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Recent Broad-Band Spectral Measurements of Turbulence in the Lower Atmosphere[1]

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Abstract. A method described by Jones and Pasquill for recording intensity of atmospheric turbulence as a function of *sampling* duration has been elaborated by Jones into a band-pass system, equivalent to specified *sampling* and *averaging* times. The principle of the method is briefly stated. Preliminary recordings of the vertical inclination of the wind in near-neutral conditions, at various heights near the ground and in a range of wind speeds, are now available. They have been analyzed to derive estimates of the standard deviation σ_w of the vertical component corresponding to the whole spectrum. The results support the constancy of σ_w with height near the ground in neutral conditions, and its linear variation with wind speed at a fixed reference height, and lead to a value of 1.3 for σ_w/u_* (u_* being the friction velocity), thereby substantially confirming an earlier estimate by Panofsky and McCormick.

Introduction. In the preceding symposium of this series, at Oxford in 1958, the author drew attention to the possibility and advantages of directly recording the intensity of atmospheric turbulence as a function of *sampling* duration, i.e., the time over which the turbulent fluctuation is observed. The concept and the practical features of an experimental system were subsequently set out in detail by *Jones and Pasquill* [1959]. Since then the system has been considerably elaborated by Jones, and a full account of this is in preparation. The present note outlines the principle of the method and presents an analysis of some preliminary recordings of the intensity of the vertical component of turbulence in neutral conditions near the ground.

The data-processing system. If a *stationary* time-lapse variation of a turbulent component is analyzed by taking running averages over times of length s, and these running averages are available over a sampling duration τ, the variance $\sigma^2{}_{\tau,s}$ that would be observed is related to the total variance (effectively infinite τ and zero s) by the equation

$$[\sigma^2_{\tau,s}] = \sigma^2_{\infty,0}\int_0^\infty F(n)\,\frac{\sin^2 \pi ns}{(\pi ns)^2}\cdot\left(1-\frac{\sin^2 \pi n\tau}{(\pi n\tau)^2}\right)dn \qquad (1)$$

$F(n)\,dn$ is the fraction of the total variance in the frequency band n to $n + dn$. The square brackets imply an ensemble average. The weighting function applied to $F(n)$ acts as a band-pass filter of the shape shown in Figure 1, with 50 per cent (power) transmission at frequencies $0.44/\tau$ and $0.44/s$. Note that, if a series of numerical filters of this type is applied, with

$$\left.\begin{array}{ll}\tau = \tau_1 & s = \tau_2\\ \tau = \tau_2 & s = \tau_3\\ \tau = \tau_{m-1} & s = \tau_m\end{array}\right\}\tau_1 > \tau_2 > \tau_3 \cdots$$

$$\sigma^2_{\tau_1,\tau_m} = \sigma^2_{\tau_1,\tau_2} + \sigma^2_{\tau_2,\tau_3} \cdots + \sigma^2_{\tau_{m-1},\tau_m} \qquad (2)$$

the first subscript referring to sampling duration and the second to averaging time. (A more detailed discussion of the effects of averaging time and sampling duration has been given elsewhere [*Pasquill*, 1961].)

The numerical filter in Figure 1 may be represented electrically, more or less closely, according to the elaboration of the electronic design. For our purposes a symmetrically shaped filter of equivalent area, with additive properties as in equation 2, has been adopted, and this is

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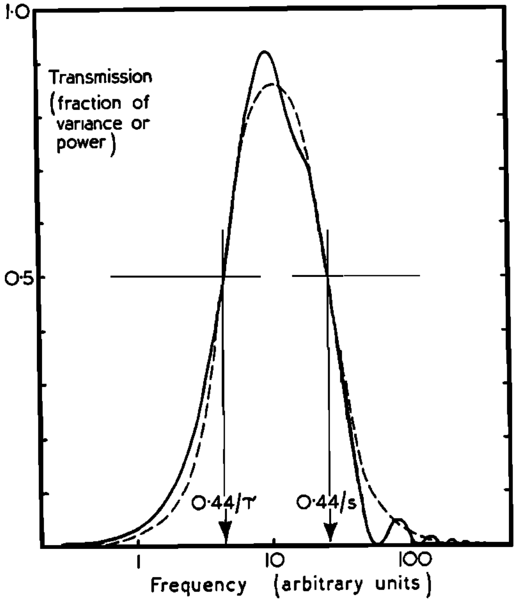

Fig. 1. Solid line is numerical filter corresponding to averaging sinusoidal fluctuations over times of length s and observing these averages over sampling duration τ, for $\tau = 6s$. Dashed line is electrical filter used as equivalent to numerical filter.

also shown in Figure 1. Numerical details for six bands are given in Table 1, n_p being the frequency for peak power transmission and $n_{1/2}$ that for 50 per cent transmission on the low-frequency or high-frequency side of the band.

Application of the method merely requires representation of the fluctuating component linearly as a fluctuating voltage, and measurement of the output of each filter. A simple technique adopted here is to rectify and smooth the output and record the resulting smoothed value of the *mean deviation* on a slow recorder. Conversion to *standard deviation* follows from multiplication by a *form factor,* which for a Gaussian distribution of fluctuations is $\sqrt{(\pi/2)}$, that is, 1.25. From a considerable number of test measurements made at Porton by Jones, in which the outputs were also squared to allow direct evaluation of the standard deviation, an average value of 1.30 has been found.

Measurement of the intensity of the vertical component near the ground. A system on the above lines is now in experimental use at Porton, giving statistical data which may be used specifically in relation to sampling and averaging times or simply as a broad-band resolution of the spectrum. Some of the earliest recordings made with it provide fairly critical estimates of the total intensity of the vertical component of turbulence in near-neutral conditions close to the ground. The term 'critical' is used in the sense that the broad-band spectral discrimination allows selection of cases for which the turbulent energy in still lower or higher frequency bands may reasonably be neglected or estimated by extrapolation.

In these recordings bands 1–4 were used with the output from a hot-wire yawmeter set up to measure the inclination of the wind. The output of the yawmeter was satisfactorily linear over the range of inclinations encountered, and its response was adequate for the range of frequencies in the bands. Fuller details of the calibration and response of the yawmeter will be included in an account of the instruments by Jones (to be published).

Recordings were also made with band 0, mainly with the yawmeter measuring the direction, but occasionally the inclination, of the wind. In the present analysis it is assumed that in this high-frequency range the intensities of the vertical and lateral components were equal, and this was certainly borne out by a few cases when direction and inclination were recorded alternately over successive intervals.

Figure 2 shows two examples of the results, chosen to illustrate the cases requiring the greatest amount of extrapolation to one side or the

TABLE 1

Band	5	4	3	2	1	0
n_p, c/min	0.06	0.36	2.2	13	78	470
$n_{1/2}$	0.025	0.147	0.88	5.3	32	192
	0.147	0.88	5.3	32	192	1152
Equivalent τ	18 min	3 min	30 sec	5 sec	5/6 sec	5/36 sec
Equivalent s	3 min	30 sec	5 sec	5/6 sec	5/36 sec	5/216 sec

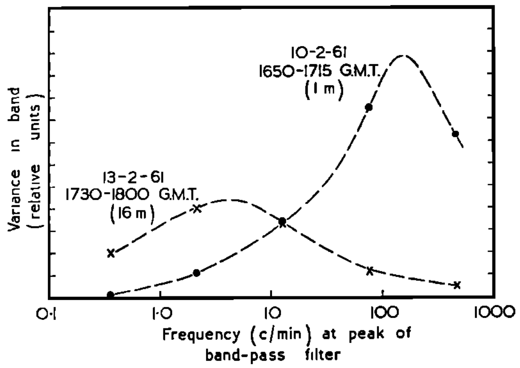

Fig. 2. Examples of results from band-pass recordings of the inclination of the wind.

other of the spectrum. No cases were accepted in which the band contributions did not contain a maximum. Estimation of the variance contribution that had been missed was made on the low-frequency side by subjective extrapolation of the observed trend, and on the high-frequency side by assuming $F(n) \propto n^{-5/3}$, according to which it is merely necessary to add 0.43 of the variance in band 0.

Acceptable results were available for 34 periods of individual duration between 15 and 60 minutes and total duration 23 hours. In 23 of them the estimated total correction for high-frequency and low-frequency omission was less than 10 per cent of the measured total variance. The maximum estimated correction was 15 per cent. After applying these corrections the resulting standard deviations of wind inclinations, σ_ϕ, were converted to standard deviations of vertical velocity, σ_w, by using the acceptable approximation for small angles

$$\sigma_w = \bar{u}\sigma_\phi$$

where $\bar{u}$ is the mean wind speed at the height of observation.

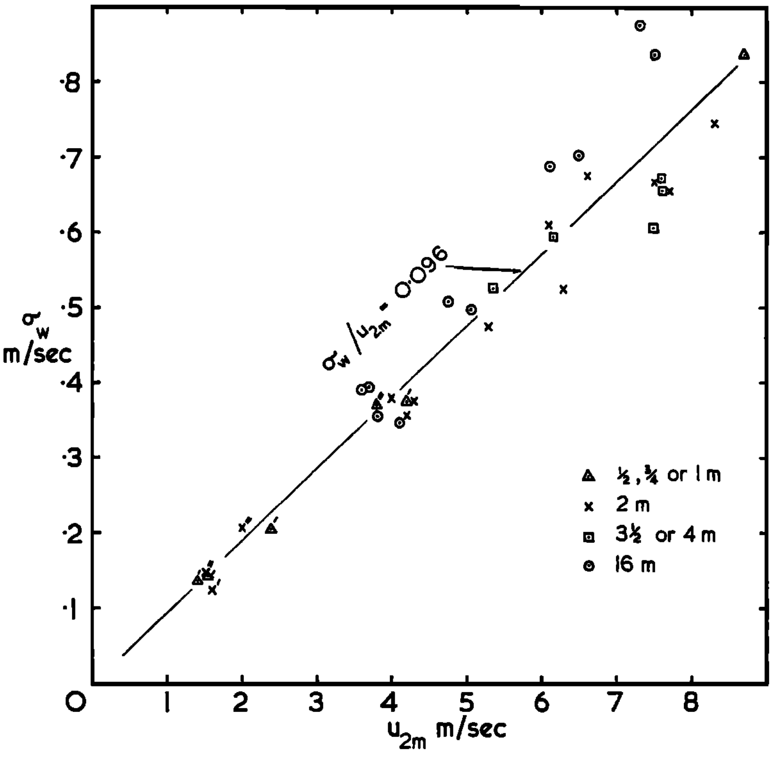

Fig. 3. Total standard deviation of the vertical component, σ_w, at various heights, as a function of wind speed at a fixed reference height of 2 meters. (The observations were made over downland, Porton, England, during January and February 1961. Stability conditions were near-neutral, the larger departures from neutral being indicated by a prime (stable) or double prime (unstable), when the stability parameter $S = \Delta T_{23\text{-}4\text{m}}/\bar{u}_{15\ 5\text{m}}^2$ in °F sec^2 m^{-2} had numerical values between 0.01 and 0.1.)

For neutral conditions of flow, dimensional argument gives

$$\sigma_w = Bu_* \qquad (3)$$

where u_* is the friction velocity and B is a numerical constant. Introducing the usual logarithmic wind profile

$$\bar{u}_z = (u_*/k) \log_e z/z_0 \qquad (4)$$

in which z_0 is the roughness parameter and k is von Kármán's constant, it follows that

$$\sigma_w/\bar{u}_z = kB/(\log_e z/z_0) \qquad (5)$$

The implication is that, in the layer in which the horizontal shearing stress is constant with height, σ_w should also be constant with height. On the other hand, for a constant surface roughness, in neutral conditions, σ_w in this layer should be directly proportional to the mean wind speed at a fixed reference height. These features have already been shown to be supported by a number of independent observations of σ_w [*Panofsky and McCormick,* 1960].

The present data, plotted in Figure 3, clearly add further support. For certain cases, which are indicated, the numerical values of a stability parameter,

$$S = \Delta T_{23\text{-}4\mathrm{m}}/\bar{u}^2_{15.5\mathrm{m}}$$

with ΔT in degrees Fahrenheit and $\bar{u}$ in meters per second, were greater than 0.01. They were, however, less than 0.1, and on the basis of previous experience do not represent markedly unstable or stable conditions. As can be seen from Figure 3 they do not display any significant departures from the trend of the remaining data.

Since the error in determining σ_w, after making the above corrections for high-frequency and low-frequency loss, seems unlikely to be more than a few per cent, a fairly critical estimate of the constant B can be made. Analysis of the data in Figure 3 by the method of least squares gives

$$\sigma_w/u_{2\mathrm{m}} = 0.096$$

and from the average of a number of wind profile observations (at heights 0.5, 2, and 15.5 meters, made in conjunction with the measurements of turbulence), $z_0 = 0.8$ cm. Substitution of these values in (5), with $k = 0.4$, leads to

$$B = 1.33$$

Previous estimates of this constant are 0.86 [*Monin,* 1959, based on Perepelkina's data], 0.7 [*Gurvic,* 1960], and 1.25 [*Panofsky and McCormick,* 1960]. From the small magnitudes of the corrections made in the evaluation of σ_w, and the insensitivity to appreciable uncertainty in the estimate of z_0, it seems unlikely that the error in the present determination of B is greater than ±0.1. The result therefore appears to confirm the highest of the earlier values.

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