Laboratory Measurements of Turbulent Diffusion in Stratified Flows[1]

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Introduction. This note consists of a brief progress report on a series of investigations into the effects of density stratification on turbulence which are being carried out at Manchester University. The work has bearing on the mixing of methane with the ventilating air in coal mines, but the results have direct application to several meteorological and oceanographic problems.

Experiments in water channels. The early experiments involved the measurements of mean quantities only, and so water was chosen as the working fluid. This enables quite high Richardson numbers to be obtained with Reynolds numbers large enough for the flow to be fully turbulent in relatively small pieces of equipment; and salt solution provides a convenient way of obtaining density differences.

In the first series of experiments [*Ellison and Turner*, 1959] the flow of a layer of heavy fluid down a slope beneath an almost stationary lighter fluid was studied; it was shown that such a flow is governed by the rate at which the moving layer entrains the ambient fluid above and that the rate of entrainment is very strongly influenced by the relevant Richardson number.

The second series of experiments were concerned with the more complicated situation of flow in a sloping rectangular pipe into which salt solution was admitted through a slit across its lowest side [*Ellison and Turner*, 1960]. The resulting flow was almost two-dimensional, and it was possible to infer values of the eddy viscosity K_M and of the eddy diffusivity for salinity K_S from measurements of the velocity and salinity profiles at two stations. The results show that the ratio K_S/K_M falls rapidly with increasing local Richardson number defined by

$$Ri = \frac{-g(d\bar{\rho}/dz)}{\bar{\rho}(dU/dz)^2}$$

This fall is, in fact, in remarkably good agreement with the predictions of a speculative theory that the author put forward some years ago [*Ellison*, 1957]. This gave

$$K_S/K_M \propto (1 - Rf/Rf_c)(1 - Rf)^{-2}$$

where Rf is the flux form of the Richardson number ($= K_S Ri/K_M$), and Rf_c is a limiting value of Rf, probably between 0.1 and 0.15. This result is of some meteorological interest, since attempts at measuring the ratio K_H/K_M (where K_H is the eddy conductivity for heat) in the atmosphere have not led to any firm conclusion about its dependence on Ri.

Experiments in wind tunnel. The success of the measurements of mean quantities emphasized the need for a study of fluctuations. There seemed little hope of measuring fluctuations in water because of instrumental difficulties, and so it was necessary to turn to air, in which hot-wire anemometers can be used to measure both velocity and temperature.

The flow chosen was the simplest which had a velocity gradient and a density gradient and for which it was possible to obtain Richardson numbers up to unity at the same time as Reynolds numbers high enough for the flow to be properly turbulent. The tunnel used has a 50 cm square section and is 10 meters long. At the entry there are gauzes to improve the flow; they are followed by a heating grid consisting of 16 horizontal tubes about 1 cm in diameter which are heated electrically in a graded manner so as to produce a linear temperature gra-

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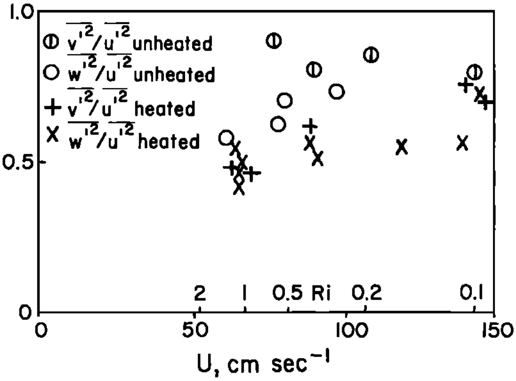

Fig. 1. Ratios of components of intensity.

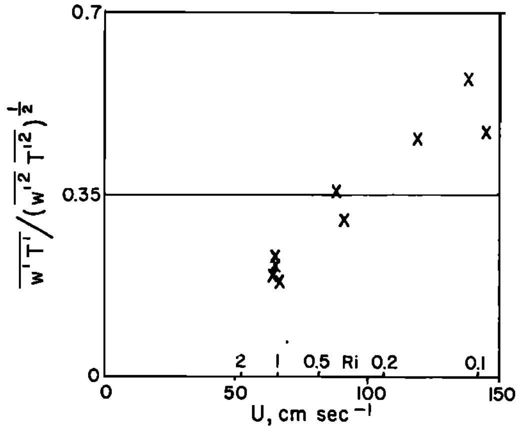

Fig. 3. Correlation coefficient of w' and T'.

dient in the air. This is followed by a further grid which has a graded drag coefficient and so produces the velocity shear. A velocity ratio of 2 : 1 is obtained in this way. The shear grid has a high blockage and produces considerable turbulence, which initially decays but later is supported by the shear.

Measurements are made of the intensities and cross correlations of the components of velocity fluctuations and the temperature fluctuations. Three wires are used simultaneously: two, which are strongly heated, are in the form of an X; and a third, which is cooler, is usually parallel to one of the others. As it proved difficult to obtain adequate calibration of the wires, their sensitivity to velocity and temperature is evaluated theoretically. For this the heat-transfer measurements of *Collis and Williams* [1959] are used in conjunction with a detailed theoretical treatment (to be published soon), including the effects of thermal conduction along the wire and predicting the frequency dependence of the response. Thus at any given frequency the output from each wire is a known linear function of the (small) fluctuations in the two components of velocity and in the temperature; and a lengthy computation performed on the Manchester University Ferranti Mercury permits measurements of the mean square output from each wire and of the mean square of the difference in output from wires taken in wires to be transformed to yield quantities of physical interest.

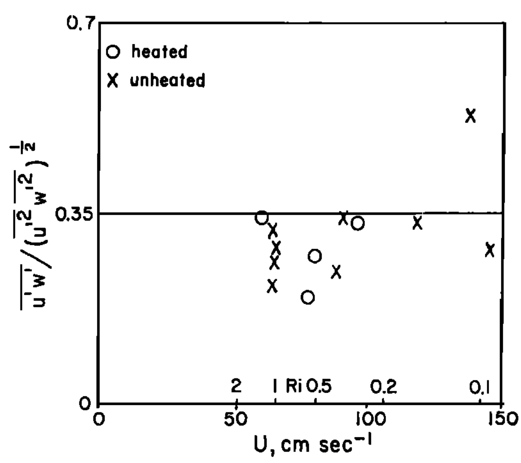

Fig. 2. Correlation coefficient of u' and w'.

At present only preliminary results are available. They were obtained closer to the shear grid than is now thought desirable, and it is likely that the turbulence still had some of the properties of decaying grid turbulence. However, the qualitative behavior of the results is believed to be correct. Unfortunately, at the low speeds (50–150 cm sec^{-1}) at which it is necessary to operate our tunnel in order to obtain high Richardson numbers, the intensity of the turbulence depends on the velocity even when there is no heat flux, and for this reason our results have been plotted against U with a second scale showing the values of Ri for the cases when the heat is switched on. Figure 1 shows the ratios $\overline{w'^2}/\overline{u'^2}$; Figure 2 shows $\overline{u'w'}/(\overline{u'^2}\ \overline{w'^2})^{1/2}$; and Figure 3, $\overline{w'T'}/(\overline{w'^2T'^2})^{1/2}$. It will be seen that the ratios of the components of the velocity fluctuations and the shear stress correlation coefficient are affected only slightly by the heat flux, although some tendency exists for the vertical and lateral fluctuations to be reduced in comparison with the longitudinal ones; on the other hand, the correlation coefficient between w' and T' is a strong function of Ri. All these results are in accordance with expectation.

Measurements at a station farther from the shear grid which include spectra of all relevant quantities are now being analyzed.

References

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