A Uniform Gradient Turbulent Transport Experiment[1]

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Abstract. This paper describes measurements in a grid-generated turbulent flow heated to a uniform temperature gradient which is self-maintained. The heating was kept small so that the experiment approximates the model of an incompressible fluid transporting a passive attribute. This model has been the subject of previous theoretical studies, which are briefly described. The measurements showed qualitative agreement with some of the hypotheses and results of the analysis given by Corrsin. Predictions of the double temperature and velocity-temperature correlations in the final period of decay have been made in the analytical work of Dunn and Reid, but measurements made in the initial period of decay are significantly different. Deductions of the Lagrangian velocity integral scale and the ratio of 'eddy' to molecular thermal conductivity are made from measurements of the temperature fluctuation and the heat-transfer correlation coefficient. They are compared with the results of the diffusion behind a line source experiment reported by Uberoi and Corrsin and also by Townsend. The development of temperature fluctuation is controlled solely by the relative magnitudes of production and decay in this experimental flow. The temperature fluctuation appears to persist in growth in all the measurements, and the cause seems to be the growth of the scale of the decaying turbulence.

1. *Introduction.* In general, turbulent transport may depend on all the derivatives of the distribution of the mean of the quantity being transported [*Corrsin,* 1957]. The same is also true of molecular transport, but in almost all such cases the contribution from the first derivative (the gradient) dominates because the scale of the transport mechanism (e.g., mean free path) is very small compared with scales formed from ratios of the gradient to the higher derivatives.

One of the lines of research on turbulent transport has adopted a flow model with a stipulated uniform temperature gradient throughout an infinite space of isotropic turbulence. In such a simplified situation the turbulent transport can depend only on the gradient of temperature, since the higher derivatives are zero by definition. The work reported herein has to do with an extension of this research on what may be called uniform gradient transport. To reduce the problem to the minimum of dynamic complexity, the previous investigators have also restricted the problem to a temperature field that has no dynamic interaction with the basic flow; that is, the heating is so small that buoyancy forces and change of fluid properties are negligible. Therefore, the results of this research are applicable to the transport of any passive attribute of the flow that obeys the conservation equation

$$\frac{\partial \Gamma}{\partial t} + U_i \frac{\partial \Gamma}{\partial x_i} = D_\Gamma \frac{\partial^2 \Gamma}{\partial x_i \, \partial x_i} \qquad (1)$$

where Γ is the concentration of the attribute, U_i is the velocity of the basic flow, and D_Γ is the molecular diffusity of the attribute Γ. Other than temperature, the attribute might be any weak concentration of a second phase in the flow, such as water vapor or weak ion concentration. Although conditions approximating uniform gradient transport of a passive attribute may exist in real flows (perhaps in geophysical situations), the dominant spirit motivating this work is the seeking of information on a turbulent transport situation in which there is the possibility of bringing together experiment and

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theory, in the hope that the basic knowledge thus gained will serve as a foundation for investigation into more complex situations.

2. *Previous work on the basic model.* *Corrsin* [1952] introduced a basic problem of turbulent transport in considering an isotropic homogeneous incompressible turbulent flow which was being swept through an imaginary transverse plane that imparted to the fluid a mean linear temperature distribution. This mean temperature profile is maintained as the fluid flows along because of the combination of homogeneity, isotropy, and linear temperature profile [*Corrsin*, 1952], for it ensures that, for every fluid particle arriving with an excess of temperature to a particular point, there is an equal probability that a fluid particle will arrive with an equal defect of temperature, and the mean temperature along a mean streamline is not altered. The latter part of this explanation was given by *Lin* [1959]. There are other ways of explaining this maintained temperature distribution, all stemming from the stipulated isotropy and homogeneity of the field (note that the characteristics of this flow depend only on the temperature gradient, which is uniform throughout the field —not on the temperature, since an incompressible fluid has been stipulated), but the explanation given above is the most elegant. Figure 1 is a schematic diagram of Corrsin's model, in which the tracks of two fluid particles (which happen to cross the same observation point at different times) are shown. This pair of tracks is representative of the fact that all the fluid particles after a long period of time or for a large number of realizations can be paired in this manner: crossing the heating plane at equal distances from the mean streamline at which they ultimately meet. In the analysis of this

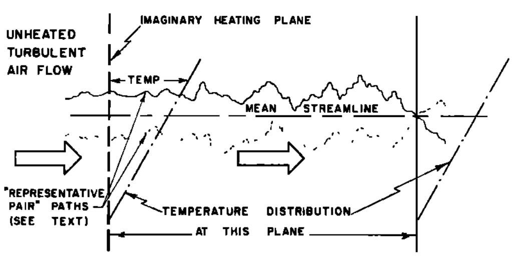

Fig. 1. Corrsin's model for the investigation of turbulent transport.

model Corrsin took the following approach: Imagine that all the statistical properties of the velocity field are known, and define 'the problem' to be the determination of the relation of the transport characteristics to the known statistical parameters of the velocity field and the given temperature gradient. Using a (Lagrangian) 'diffusion by continuous movements' analysis in the manner of *Taylor* [1921] but in reverse, and a combined Lagrangian-Eulerian approach, he was able to derive some of the relations that will be used in section 12. Two of the important results of this work were the demonstration that an asymptotic region of heat transfer and temperature fluctuation developed despite the singular heating plane, and the occurrence of the Lagrangian integral scale in the place occupied by the mixing length in the traditional treatments of turbulent transport phenomena. However, Corrsin did not attempt to estimate the spatial structure of the velocity-temperature correlation or the temperature-temperature correlation. His analysis is restricted to stationary, isotropic turbulence. He also assumed local isotropy of the temperature field. Moreover, he arrived at his results with the speculative hypothesis (due to *Taylor* [1935]) that turbulent and molecular transport processes are statistically independent; later work has shown that the two processes are not independent [*Saffman*, 1960]. Thus the theoretical problem is by no means completely solved. The temperature-gradient vector in this model give the statistical aspects of the temperature and temperature-velocity correlation fields an axisymmetrical nature. *Melese* [1954] worked out the simplifications of the governing equations due to axisymmetry. He applied his work to the uniform gradient transport model and calculated the heat transfer (one-point) correlation for the restricted case of negligible convective acceleration terms in a nondecaying isotropic turbulence.

A complete solution of the linearized case (heat transfer in isotropic turbulence during the final period of decay) was worked out by *Dunn and Reid* [1958]. Their results give a complete Eulerian description of the velocity-temperature spatial correlation and also the two-point temperature correlation. This information is valuable in approaching the more important problem of transport in a fully turbulent medium, that is, where the nonlinear terms are operating. Since a satisfactory theory does not exist, experiments like those reported here are required.

3. *Present project objectives.* The aims of this project were (1) to see how well this ideal model with turbulent transport could be approximated in the laboratory; (2) to investigate the turbulent production of temperature fluctuation, especially the development of a quasi-asymptotic state, if any; (3) to measure the heat transfer correlation coefficient; (4) to measure the spatial structure of some of the pertinent correlations, in particular to ascertain what range of eddy size transports most of the energy.

4. *The experiment.* The main features of the test section in which the turbulent transport flow model was created is shown schematically in Figure 2. A large initial advantage accrues to the selection of a grid-generated turbulence as the basic flow pattern, because measurements in this flow have played an important role in answering basic questions about turbulence dynamics and hence a great deal of information is available [*Batchelor,* 1956; *Hinze,* 1959]. The popularity of this flow is due to its homogeneity in directions lateral to the flow, approximate homogeneity in the flow direction, and approximate isotropic properties. Moreover, the uniform mean flow and low turbulence levels permit relatively reliable turbulence measurements.

Figure 3 indicates the success achieved in heating the air to a uniform temperature gradient and its ability to maintain itself to 72 mesh

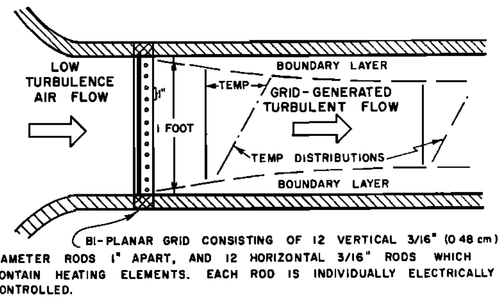

Fig. 2. Laboratory model for the study of turbulent transport.

lengths (M, mesh length = 1 inch). Actually the uniform temperature gradient was found to be maintained to 140 mesh lengths except for the encroachment of the thickening boundary layers. These mean temperature measurements were made with a rake of 11 thermocouples with 1-inch spacing. Check measurements made in between the positions shown in Figure 3 indicated that the profiles were not far from the straight line on the graphs.

If the heating of the air in this test section is to serve only as a passive attribute, the heating should not alter the turbulence dynamics. Figure 4 provides evidence indicating that the heating did not alter the longitudinal velocity fluctuations in addition to showing how uniform these fluctuations are across the test section (the waves in the traverse curves indicate the small amount of residual unevenness due to the fact that the turbulence field is generated by the rods of the grid which are spaced 1 inch

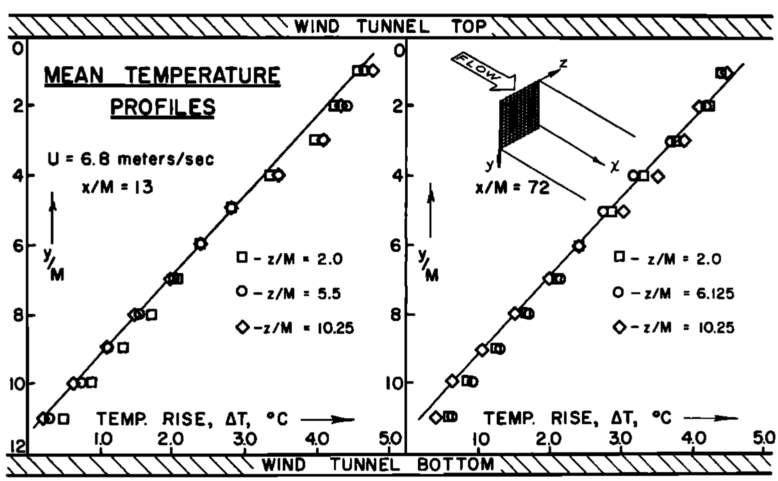

Fig. 3.

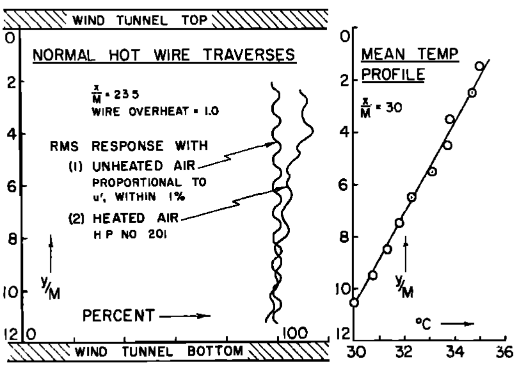

Fig. 4.

apart). A single hot wire responds to both velocity and temperature fluctuations, and the ratio of the responses varies with the overheat (temperature rise above ambient) of the wire. As is indicated in Figure 4, with the small amount of temperature fluctuations in the unheated air the hot-wire response at an overheat of 1.0 is proportional to the longitudinal velocity fluctuations to within 1 per cent. However, the increased temperature fluctuation in the heated air increases the response of the hot wire. Calculations using the level of velocity and temperature fluctuations from independent measurements and the sensitivities of the hot wire show that any alteration of the velocity fluctuations due to heating the air is quite small. The designation 'H.P. no. 201' refers to the heating program number. Heating program is the phrase used to denote the set of voltages applied to the heating elements in the rods to achieve a desired temperature distribution.

The fact that the difference of the two response curves in Figure 4 is not uniform across the test section is due to the nonuniformity of the temperature fluctuations at this streamwise station. This aspect of the flow is demonstrated more clearly in Figure 5. These curves are representative of the development of the temperature fluctuations in the streamwise direction. The explanation of this development may be as follows: (1) the rms temperature fluctuation produced in the wake of a heated rod is proportional to the mean temperature rise of that rod; (2) when the turbulence has become homogeneous, the rms temperature fluctuation would be approximately linear across the test section; (3) the change in the initial linear distribution of rms temperature fluctuation is due to the competing processes of decay and production; (4) the decay of the temperature fluctuation is proportional to the level of temperature fluctuation [*Mills, Kistler, O'Brien, and Corrsin*, 1958]; (5) the rate of production of temperature fluctuation (which is roughly a function of the local temperature gradient) is constant across the test section; (6) the resultant tendency is for the temperature fluctuation distribution to approach homogeneity across the test section, as can be seen in Figure 5. The irregularities in these curves are due to imperfections in the experimental situation: the air entering the open-end wind tunnel had its own temperature distribution, which was subject to slow nonuniform changes, and the air heating system did not have sufficient fineness of control to permit precise final temperature distributions to be obtained. Although the rms temperature fluctuation (θ') distribution at $x/M = 102$ in Figure 5 shows evidence of having been formed by the presence of the temperature gradient, the middle part is still decreasing from the measurements upstream. The ambiguity shown here was also found in a similar set of measurements, and it was decided to extend the test section another 40 mesh lengths to obtain more evidence on the development of θ'.

Figure 6 shows the results of measurements farther downstream. θ' at $x/M = 140$ has increased over the distribution in the y direction, and it was concluded that the production of temperature fluctuation due to the presence of the mean temperature gradient was playing the dominant role in the development of the flow between $x/M = 102$ and 140.

However, at these distances from the grid the velocity fluctuation becomes low and the bound-

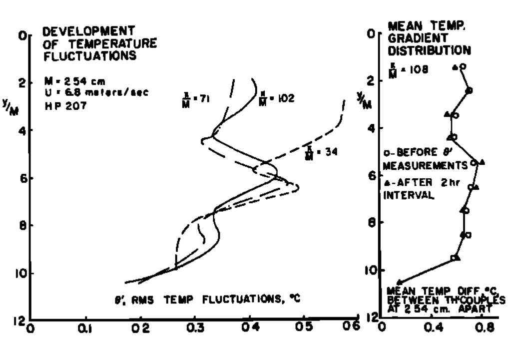

Fig. 5.

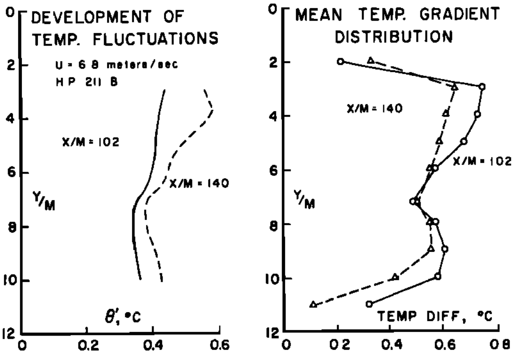

Fig. 6.

ary layers become thick. In order to improve on these conditions an additional grid (biplanar, 3/16-inch rods on 1-inch mesh) was placed at $x/M = 72$. This secondary grid, which was not heated in any way, proved to be a great success, because besides increasing the intensity of velocity fluctuation (making measurements more reliable) it also thinned the boundary layers, all without impairing the uniformity of the temperature gradient and the θ' distributions as shown in Figure 7. Final measurements of the correlations were made at the two stations $x/M = 102$ and 140 with and without the use of the secondary grid at $x/M = 72$. All the measurements were made with a mean air speed of 6.8 m/sec and a velocity fluctuation level of the order of 1 per cent without the secondary grid and 2 per cent with the secondary grid in place. All the correlation measurements were made in the presence of θ' of order 0.35°C and a temperature gradient of about 0.25°C/cm. (There were small variations in these values for each measurement, owing to adjustment of the heating.) The grid Reynolds number (UM/ν) was 11,500, and the turbulent Reynolds number $(u'\lambda/\nu)$ was approximately 30.

5. *Hot-wire techniques.* The final hot-wire measurements were made with the Hot-Wire Anemometer Constant Current System manufactured by Shapiro and Edwards. The techniques used were largely adaptations of the methods described in the basic references on the subject [*Kovasznay*, 1954; *Corrsin*, 1949; *Morkovin*, 1956]. The measurement of temperature fluctuation of small magnitude with the hot wire (for this application the 'hot wire' is really a resistance thermometer) is complicated by the requirement of a low wire current so that the response of the wire will be negligibly affected by the velocity fluctuation. Sufficient signals can be obtained with the use of a high-resistance wire. If the decision is made to maintain the simple straight arrangement of the wire, high resistance cannot be attained by increasing the wire length indefinitely, because the instrument will increasingly lose response to the small-scale fluctuation. As a consequence, finer wire is required, and for some of the measurements reported in this paper it was necessary to use the finest wire available: 0.000025-inch diameter. This wire is prepared by the Wollaston process and can be obtained from the Sigmund Cohn Co. It is available in two materials: pure platinum, and an alloy consisting of platinum with 10 per cent rhodium. An additional advantage of this fine wire is that its thermal lag is so small that for the spectral content of the signals in this experiment the usual electrical compensation was not necessary.

The two-point temperature correlation measurements were made with the aid of the switching circuits of the Shapiro and Edwards System. The two-point velocity-temperature correlation measurements were also done with the switching circuits; these measurements are more difficult to make because they require an instrument response to the lateral velocity fluctuation. This response is obtained from the well-known arrangement of two wires mounted in an X array. The difference in the response of the two wires when the instrument is properly made and mounted is proportional to the lateral velocity fluctuation and furthermore is independent of the temperature fluctuation. The two-point ve-

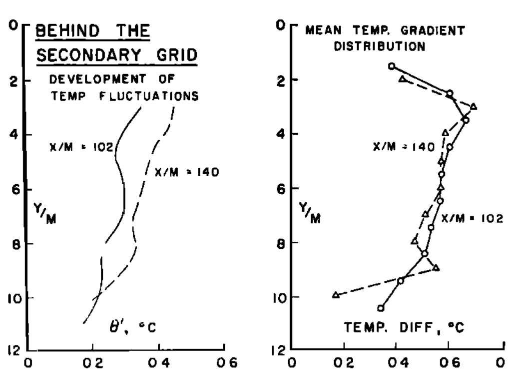

Fig. 7.

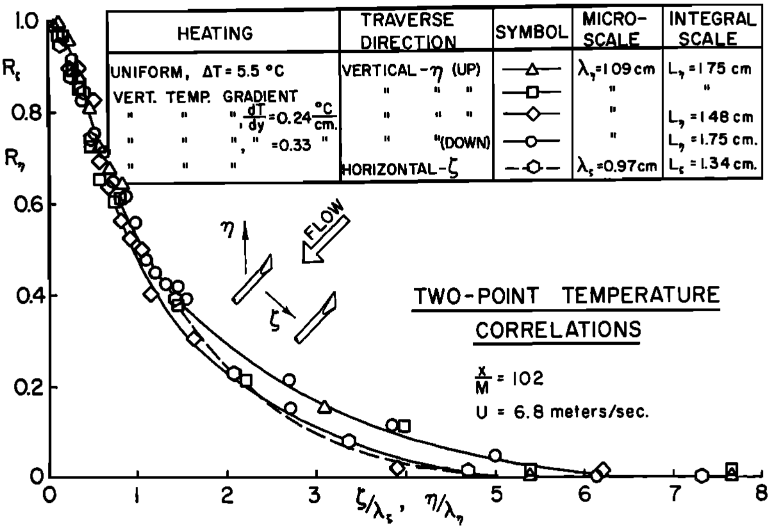

Fig. 8.

locity-temperature correlation measurement was made with the X probe and a third wire mounted on a second probe designed to respond solely to the temperature fluctuation. The second probe was traversed to obtain the spatial correlation graphs.

The correlation measurements were considerably simplified by making the appropriate measurements to permit the direct determination of a dimensionless coefficient. This technique obviated the necessity of calibrating all the wires used in the correlation measurements. The dimensional value of the correlation can be obtained from the dimensionless coefficient and separate measurements of v' (rms lateral velocity fluctuation) and θ'. The measurements of θ' were done with the usual calibration procedures.

The limitation of experimental time prevented the perfecting of the X-probe technique. The X probes used produced a vibration of approximately 1500 cps in the flow. Fortunately the significant 'energy' in the velocity and temperature fluctuations at the air flow used is below 1000 cps, so that a simple solution was possible since the unwanted 1500-cps signal could be filtered out. However, the resulting response did lose 'information' about the small-scale fluctuation, and this fact is reflected in the velocity-temperature spatial correlation graphs.

6. *The two-point temperature correlation measurements.* These measurements are shown in a series of graphs starting with Figure 8. Figure 8 is a composite of the two-point correlations found with a uniformly heated grid, and with a grid heated to produce a temperature gradient—in the latter case both for traverse along and normal to the direction of the temperature gradient (appears as vertical and horizontal, respectively, in Fig. 8). Interestingly, the correlation curves were almost identical for both methods of heating the air; it was surprising to find that they differed little for traversing along and normal to the gradient in the temperature gradient heated air. The expectation was for a significant difference due to the bias introduced by the presence of the temperature gradient. The two full curves shown in Figure 8 are an

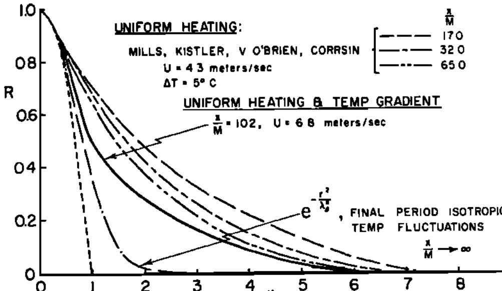

Fig. 9. Comparative two-point temperature correlations.

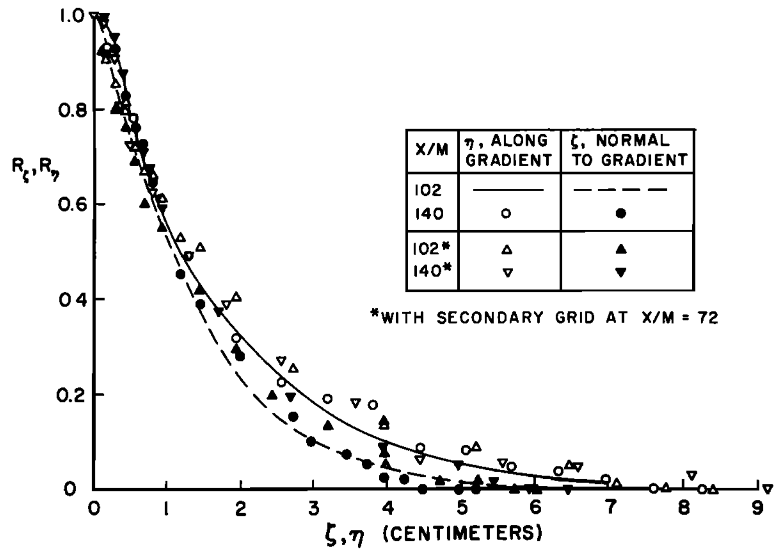

Fig. 10. Two-point temperature correlations.

indication of the reproducibility of these data with our experimental arrangement.

A comparison of these measurements with the correlations obtained by *Mills, Kistler, O'Brien, and Corrsin* [1958] is shown in Figure 9. They made these measurements with a biplanar grid consisting of horizontal and vertical heating rods, $\frac{1}{4}$ inch in diameter and with $M = 1$ inch. The consistency in the progression of the correlation curves toward the limiting case of the Gaussian (theoretical result) [*Corrsin*, 1951] supports the validity of the measurements. λ_θ/M (λ_θ is the microscale associated with the two-point temperature correlations) is 0.43 for the correlation obtained by traverse along the gradient and 0.38 for the traverse normal to the gradient (from Fig. 8). Extrapolation of the Mills data shows a $\lambda_\theta/M = 0.40$ at the same streamwise station.

All the two-point temperature correlation measurements that were made are presented in Figure 10. In these data, the abscissa was kept in dimensional units rather than normalizing by the respective microscales, the determination of which is subject to error. That the data show a definite trend for higher correlations for the traverse along the gradient is reflected by the integral scales presented in Table 1. The difference between the correlations along and normal to the gradient agrees, in tendency, with the results of Dunn and Reid. However, their linearized theory shows a considerably larger difference between the two; the one normal to the gradient has a negative portion that has a minimum of −0.13. Compared with the Dunn and Reid calculated correlations, the present measurements show slight difference between the two correlations. The similarity between the correlation measurements might be the manifestation of the more active mixing in the grid-produced turbulence as compared with the 'infinitesimal' fluctuations in the linearized model of turbulence dealt with by Dunn and Reid.

7. *The two-point velocity-temperature correlations.* Graphs of the two-point velocity-temperature spatial correlations are shown in Figures 11 and 12. The velocity referred to here is always the velocity component along the gradient. It is this component that is associated with the turbulent transport of heat. Figure 11 singles

TABLE 1. Integral Scales (L_θ/M) for the Two-Point Temperature Correlations (Fig. 10)

X/M	Traverse along Gradient	Traverse Normal to Gradient
102	0.69	0.55
140	0.69	0.54
102*	0.69	0.59
140*	0.72	0.65

* With secondary grid at $X/M = 72$.

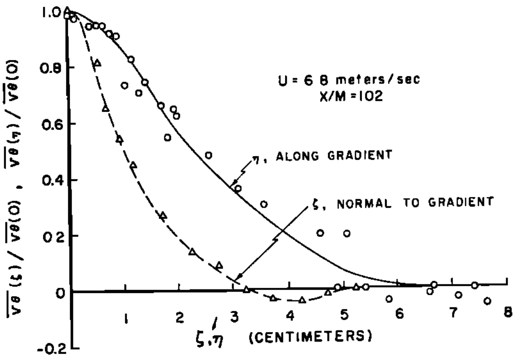

Fig. 11. Two-point velocity-temperature correlations.

out the pair of correlations (traverse along and normal to the gradient) measured at $x/M = 102$ without the use of the secondary grid. This pair shows the essential characteristics of the correlations as measured in this experiment. The scatter of the measurements is largely due to the fact that the data were obtained over two days with different wires.

An interesting comparison can be made with the graphs in Figure 11. These curves resemble the two-point velocity correlations in isotropic turbulence [*Batchelor,* 1956, p. 48; *Hinze,* 1959, p. 150]. The similarity between the velocity-temperature and double velocity (all velocities in the direction of the gradient) correlations seems reasonable inasmuch as the one-point temperature-velocity correlation coefficient is relatively high, approximately 0.5; the measurements of this quantity will be presented in the next section. The *Dunn and Reid* [1958] calculations for these correlations show similar results except that the correlation in the direction normal to the gradient has a more pronounced negative correlation region: the maximum negative correlation is -0.13.

Figure 12 shows all the velocity-temperature correlation measurements. They exhibit some scatter, but most of the data points show a systematic variation along the lines displayed in Figure 11. Table 2 presents the integral scales obtained from faired curves through the data plotted in Figure 12. As was mentioned in the comments on hot-wire technique, the oscillation introduced by the X probe required attenuation of the high frequencies, and consequently the structure of the correlation curves for small separation is much too flat and reliable deductions of the microscale cannot be expected.

8. *Other measurements.* An experiment was performed in which for a given heating program the total power to the grid was varied. The results are shown in Figure 13. The curve in the left-hand graph is probably due to heat loss from the grid at the rod ends. The nonzero intercept in this graph occurs because of the small temperature gradient in the unheated air. The right-hand curve shows that, up to the magnitude of temperature gradient achieved on the two days of the test, θ' was proportional to the magnitude of the temperature gradient, dT/dy. This linearity appears to be excellent proof of the passivity of the temperature fluctuation for this level of heating. These data accurately determine the ratio $\theta'/(dT/dy)$. However, full understanding of the processes involved in the establishment of the temperature fluctuation requires measurements of the development of these fluctuations in the streamwise direction. Such information is given in Figures 5 and 6, but another method of measurement gave the results shown in Figure 14. These graphs show a general rise of the mean square temperature fluctuation in the streamwise direction along the mean streamline in the center of the test section with an air flow heated to a temperature gradient. The full curve and the dashed curve are plotted on different scales, which is required because of the variability of conditions in the wind tunnel on different days. The decay of the mean square temperature fluctuation with a uniformly heated grid is also shown in this graph. Comparison of the graphs shows the dominant role played by the temperature gradient in producing temperature fluctuation. These

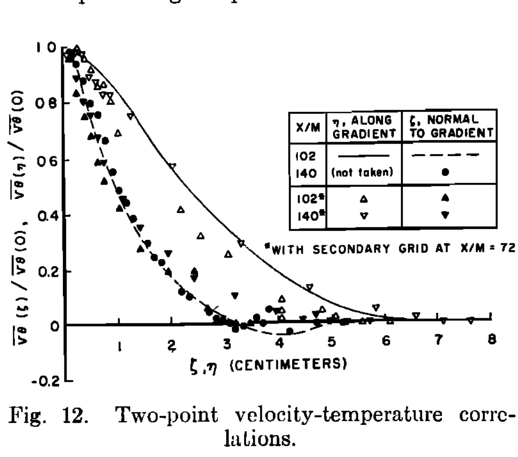

Fig. 12. Two-point velocity-temperature correlations.

TABLE 2. Integral Scales ($L_{\overline{v\theta}}/M$) for the Two-Point Velocity-Temperature Correlations (Fig. 12)

X/M	Traverse along Gradient	Traverse Normal to Gradient
102	1.00	0.44
140	(not taken)	0.46
102*	0.83	0.40
140*	1.01	0.52

* With secondary grid at $X/M = 72$.

characteristics of this flow are especially interesting when it is recognized that Corrsin's theory suggests that interpretation of these data with Lagrangian scales might be fruitful (see section 12).

A quantity of prime interest in this work is the coefficient of correlation between the fluctuations of the velocity component along the gradient and the temperature fluctuations at the same point, the 'heat transfer correlation.' As was pointed out in section 5, this measurement is the most difficult of those encountered in our work. Consequently within the spirit of the pilot project there was only sufficient time for a few measurements of this type—not enough for a precise determination. The most reliable determination yields a heat transfer correlation coefficient of

$$R_{\overline{v\theta}} = 0.48$$

These measurements were made at $x/M = 102$.

9. *The ratio of 'eddy' to molecular thermal conductivity.* With the measurements presented above, it is possible to calculate the ratio of effective or 'eddy' thermal conductivity in the turbulent flow to the molecular thermal conductivity. As a consequence of the definitions involved this ratio can be expressed as follows:

$$\frac{K_T}{K} = \frac{\rho C_p \overline{v\theta}/(dT/dy)}{K} = \frac{R_{\overline{v\theta}} v' \theta'}{\alpha (dT/dy)} \qquad (2)$$

where

K_T is eddy thermal conductivity.
K is molecular thermal conductivity.
ρ is the density of the fluid.
C_p is the specific heat of the fluid.
$\overline{v\theta}$ is the heat transfer correlation.
$R_{\overline{v\theta}}$ is the heat transfer correlation coefficient.
v' is the rms velocity (component along the gradient) fluctuations.
α is the molecular thermal diffusivity $= K/\rho c_p = 0.21$ cm²/sec for air.

Using the values reported above, $\theta'/(dT/dy) = 1.41$ cm from Figure 13 and $v' = (0.0078)(680)$ cm/sec from measurement yields $K_T/K = 17$ at $x/M = 102$ without the secondary grid in the wind tunnel. This value is about ½ of that obtained by interpolating the data of Uberoi and Corrsin far behind a line source of heat at comparable x/M and R_λ [*Uberoi and Corrsin*, 1953, Fig. 38]. It is surprising that turbulence of such low intensity is such an effective transport agent.

The important ratio K_T/K can be seen to be the product of $R_{\overline{v\theta}}$ and a dimensionless quantity that has the form of a Péclet number, since $\theta'/(dT/dy)$ is a length. If this ratio is small the molecular diffusion influences the total transport in a turbulent flow; if it is large the total transport is expected to be largely independent of molecular processes [*Saffman*, 1960]. The interaction of molecular and turbulent processes is only partially understood, and it appears that the present experimental model with the possibility of variation of the ratio K_T/K might be a useful tool for further investigation of this interaction phenomenon.

10. *The development of the mean square temperature fluctuation.* A conservation equation for the mean square temperature fluctuation, $\overline{\theta^2}$, can be obtained from the equation for the conservation of thermal energy [*Corrsin*, 1952*a*]: equation 1 with Γ considered to be temperature. The general equation for $\overline{\theta^2}$ can be specialized for application to the flow under discussion by considering a steady-state flow with a nonvarying mean flow, U, oriented in the x direction, and assuming homogeneity in the direction of the heating rods (z direction). The specialized equation becomes

$$\underset{(a)}{U\frac{\partial \overline{\theta^2}}{\partial x}} = \underset{(b)}{-2\overline{\theta v}\frac{dT}{dy}} - \underset{(c)}{2\alpha\overline{\left(\frac{\partial \theta}{\partial x_i}\right)\left(\frac{\partial \theta}{\partial x_i}\right)}}$$

$$+ \alpha\left(\underset{(d)}{\frac{\partial^2 \overline{\theta^2}}{\partial x^2}} + \underset{(e)}{\frac{\partial^2 \overline{\theta^2}}{\partial y^2}}\right) - \underset{(f)}{\frac{\partial}{\partial x}\overline{u\theta^2}} - \underset{(g)}{\frac{\partial}{\partial y}\overline{v\theta^2}} \qquad (3)$$

The significance of each term to the conservation

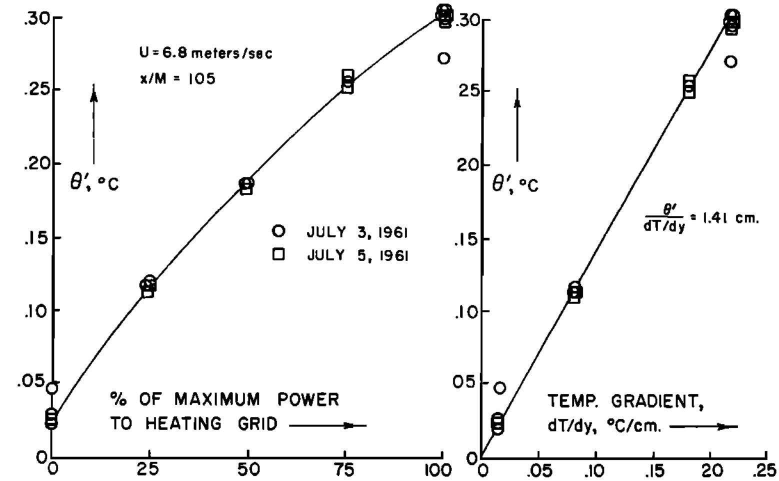

Fig. 13. Temperature fluctuation versus temperature gradient at a point.

of the fluctuation can be considered to be the following: (*a*) mean flow convection or growth; (*b*) production from the mean temperature field (always positive); (*c*) decay through molecular action on the local temperature gradients (always negative); (*d*) and (*e*) molecular transport in the presence of second gradients of $\overline{\theta^2}$; (*f*) and (*g*) turbulent transport of $\overline{\theta^2}$.

Homogeneity in the y direction, which is approximately realized in this experiment, makes terms (*e*) and (*g*) vanish. The changes in the streamwise direction are small enough so that terms (*d*) and (*f*) are negligible compared with the remaining terms:

$$U \frac{d\overline{\theta^2}}{dx} = -2\overline{\theta v} \frac{dT}{dy} - 2\alpha \overline{\left(\frac{\partial \theta}{\partial x_i} \frac{\partial \theta}{\partial x_i} \right)} \tag{4}$$

If it is assumed that the temperature fluctuation field is locally isotropic (and the results of the measurements of the spatial correlations make such an hypothesis reasonable), the decay term can be simplified to the form $-12\ \alpha(\overline{\theta^2}/\lambda_\theta{}^2)$ [cf. *Corrsin*, 1951].

Using the measurements of this experiment it is found that the production term has the magnitude 0.42 (°C)^2/sec, and the decay term 0.28 (°C)^2/sec. These values predict a growth rate of 0.14 (°C)^2/sec in the streamwise direction. The graphs in Figure 6 give measurement evidence on the average growth rate between x/M = 102 and 140. Excluding the top region, which is probably influenced by boundary-layer fluctuations, these data indicate that the growth rate varied from a minimum of 0.13 to a maximum of 0.26 (°C)^2/sec across the test section.

11. *Comparison of the experiment and the theoretical model.* Theoretical models for the study of turbulent flow usually consider fluids of constant density, and this was assumed in *Corrsin*'s [1952*a*] investigation. In a real fluid, however, there will be density fluctuation associated with the temperature fluctuation, and it is important to determine the significance of this aspect of the flow in the wind tunnel.

One effect of the density fluctuations is to cause buoyancy forces, and the question arises whether these forces affect the energy content of the turbulent motion. If the heating is to be passive, the 'buoyant' energy should be small compared with the other energy terms. The nature of this flow permits the same approxima-

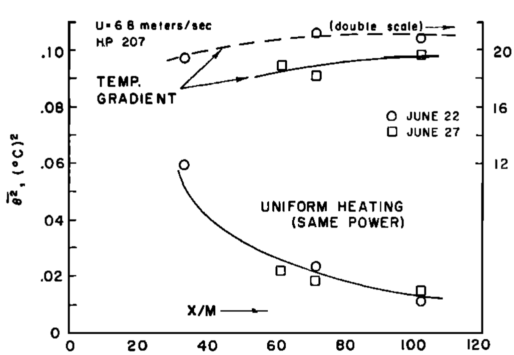

Fig. 14. Development of mean square temperature fluctuation in streamwise direction.

tions as were employed by *Townsend* [1958], and hence his turbulent energy equation can be applied here in the form specialized for this flow:

$$U\frac{d}{dx}(\tfrac{1}{2}\overline{q^2}) + \frac{d}{dx}\left(\tfrac{1}{2}\overline{q^2u} + \frac{1}{\rho}\overline{pu}\right)$$
$$= \frac{\nu}{2}\frac{d^2\overline{q^2}}{dx^2} - \nu\overline{\left(\frac{\partial u_i}{\partial x_K}\frac{\partial u_i}{\partial x_K}\right)} + \frac{g}{T}\overline{v\theta} \qquad (5)$$

The new symbols introduced in this equation are:

$\overline{q^2} = \overline{u_i\, u_i}$, proportional to turbulent kinetic energy.

p = pressure.

g = gravitational acceleration.

'Buoyancy' energy is represented by the third term on the right. The correlation is negative, which means that the density fluctuations drain energy from the turbulent energy. The flow is hotter and lighter on top and the turbulent motion is partly correlated with the density fluctuations, so that energy is required to drive lighter blobs of fluid down against the action of gravity and to drive heavier blobs up also against the action of gravity.

The dominant term in equation 5 for grid-generated turbulence [*Corrsin*, 1952*b*] is the second viscous decay term on the right-hand side, which for approximation purposes can be written $-15\ \nu\ \overline{u^2}/\lambda^2$, the result for isotropic turbulence. For quantities characteristic of the flow described in this work it is found that the 'buoyancy' energy drain is only 1.5 per cent of the viscous decay term. This supports the empirical check (mentioned earlier) on the passivity of the thermal tagging.

The variation of the physical properties of the medium with temperature can seriously affect the analysis of turbulence [*Tritton*, 1961]. A detailed study of these matters as they pertain to the theory and measurements in the model described in this work might be useful.

12. *Comparison of results with previous findings. Corrsin*'s [1957] analysis yields the following asymptotic results for the nondecaying turbulent flow of a fluid with no molecular diffusivity:

$$U(d\overline{\theta^2}/dx) = 2v'\mathcal{L}(dT/dy)^2 \qquad (6)$$

$$\overline{\theta v} = -v'\mathcal{L}(dT/dy) \qquad (7)$$

$\mathcal{L}$ is a transverse Lagrangian scale which is equal to $(v'/U)\ L_v$. L_v is the Lagrangian lateral velocity integral scale in the x direction which is derived from the true Lagrangian time correlation by application of Taylor's hypothesis.

The above two equations agree with the relationship afforded by the Eulerian equation for $\overline{\theta^2}$ when $\alpha = 0$ (see equation 4). The identity of the Lagrangian and Eulerian results for this case is an important aspect of Corrsin's investigation. However, as equation 6 shows, $\overline{\theta^2}$ will constantly grow—the consequence of considering a fluid with zero molecular diffusivity. Therefore the heat transfer correlation coefficient for this case has an asymptotic value of zero, since $\overline{\theta v}$ does reach a finite asymptote.

For a real fluid with finite molecular diffusivity the temperature fluctuation will decay and the rate of decay will increase with increasing $\overline{\theta^2}$. Hence an equilibrium will be reached between the production and decay of $\overline{\theta^2}$. This balance and the assumption of isotropy permitted Corrsin [1952*a*] to arrive at an expression for the asymptotic level of $\overline{\theta^2}$:

$$\overline{\theta^2} = -(\lambda_\theta^{\,2}/6\alpha)\overline{\theta v}(dT/dy) \qquad (8)$$

Now it is possible to obtain a finite result for the asymptotic $R_{\overline{v\theta}}$ by invoking the argument that the turbulent and molecular processes are statistically independent. The result is

$$R_{\overline{v\theta}} = \overline{\theta v}/\theta' v' = (6\alpha\mathcal{L}/v'\lambda_\theta^{\,2})^{1/2} \qquad (9)$$

Some of the steps of the above analysis can be compared with the results of the measurements of this experiment. Figures 8, 9, and 10 provide supporting evidence for the assumption of isotropy of the temperature fluctuation. Figure 14 indicates a tendency for $\overline{\theta^2}$ to reach an asymptotic level, but the fact that the measurements deal with a decaying turbulence complicates this comparison. If, for a first approximation, the expressions derived by Corrsin for a nondecaying turbulence are applied to the decaying grid-generated turbulence, the measurements in the experiment can be interpreted by taking account of the variation of the quantities appearing in Corrsin's expressions according to their known behavior in the decaying turbulence.

To elucidate the above statement consider $\overline{\theta v}$. *Uberoi and Corrsin* [1953] and also *Townsend* [1954] found that this quantity became asymptotically constant in the diffusion from a line source in grid-generated turbulence. The same behavior is expected in the present experiment, since the physical processes determining $\overline{\theta v}$ in the line source experiment and in the turbulent flow heated to uniform temperature gradient are similar. Equation 7 can be applied to the grid-generated turbulence even though it was derived for a nondecaying turbulence by employing the relation for the streamwise variation for v' in the initial period of decay behind a grid. In addition it is assumed that $\mathcal{L}$ varies as other scales in the turbulence. Then the product $v'\mathcal{L}$ is expected to remain constant in the initial period of decay of the turbulence, and hence $\overline{\theta v}$ as indicated by (7) is also expected to remain constant.

Another method, employing a more sophisticated analysis, yields the same result: $\overline{\theta v}$ can be considered equal to

$$\frac{1}{2}\left(\frac{d}{dt}\,\overline{Y^2}\right)\frac{dT}{dy}$$

[*Corrsin*, 1952*a*], and it has been shown [*Batchelor and Townsend*, 1956], with the aid of a self-preservation hypothesis employing velocity and time scale variations in the flow, that $(d/dt)\,\overline{Y^2}$ approaches a constant in the initial period of decay. $\overline{Y^2}$ is the mean square dispersion of a fluid particle, and this argument is applicable in so far as the molecular diffusion is negligible.

Expression 8, with the interpretation outlined above, suggests that the asymptotic variation of $\overline{\theta^2}$ would be a linear increase with x, if λ_θ behaves as it does behind the uniformly heated grid [*Mills, Kistler, O'Brien, and Corrsin*, 1958]. In fact, using the relation from this case, $\lambda_\theta = \sqrt{2\alpha/\nu}\,\lambda$, and the empirical relation $\lambda^2 = (10\nu/U)(x - x_0)$, which has been found for the initial period of decay of the turbulence, an alternative calculation for the growth term ((*a*) of equation 3) can be made; the result is

$$U(d\overline{\theta^2}/dx) = -10/3\overline{\theta v}(dT/dy) \qquad (10)$$

Of course, the above growth rate is appropriate to the asymptotic state, whereas the growth term previously discussed (section 10) is appropriate to the flow before the asymptotic state is reached. Using the relation above and the same data discussed in section 10, the asymptotic growth rate is predicted to be 0.70 (°C)²/sec. This result is much higher than anything measured. The data of Figure 14 also disagree with the notions expressed above, because the $\overline{\theta^2}$ curves for temperature gradient heating would have to extrapolate back linearly to an intercept on the x/M axis at approximately $x_0/M = 10$ in accordance with the behavior of λ in the initial period of decay [*Batchelor*, 1956]. These discrepancies are not surprising, since the considerations discussed above in section 4 showed that with the present method of heating the air other processes (in addition to the one under study) are involved for a good part of the test section. Therefore an asymptotic state could not become manifest until measurements were made farther downstream. Such a prospect is not promising, because the asymptotic temperature fluctuation state envisaged here depends on the maintenance of the initial period of decay, and the present measurements were made up to the limit of this region when the secondary grid was not used. Unfortunately, measurements far downstream of the secondary grid have not been made as yet.

A fundamental component of the relations derived by Corrsin is the Lagrangian velocity integral scale. This quantity has not been directly measured. Measurement by the indirect method of using the heated wake from a line source perpendicular to the direction of flow has been reported by *Uberoi and Corrsin* [1953]. They followed Taylor in assuming that in the decaying grid-generated turbulence the Lagrangian correlation coefficient is only a function of

$$\eta = \int_{x_1}^{x} \frac{v'(x)}{U}\,dx$$

A consequence of this transformation is the prediction that for long diffusion times the mean square dispersion is proportional to η, which in turn is proportional to $(x - x_0)^{1/2}$. However, Townsend showed that the data on the diffusion behind a line source fared better with a variation of mean square dispersion proportional to $(x - x_0)$. He also indicated that this behavior is consistent with the demonstrated similarity and self-preservation in the initial period of decay. Nevertheless, Uberoi and Corrsin present values of the Lagrangian scale computed from their measurements. Townsend did not compute this quantity, because he considered that the influence of the molecular diffusion did not permit a clear interpretation of his data in this manner.

An alternative approach to the determination of the Lagrangian integral scale is to relate it theoretically to the Eulerian scale, which is directly measurable. A complete relation has not been described, but *Deissler* [1959] has shown in an ad hoc way that Lagrangian correlations are approximately equal to Eulerian correlations in low-Reynolds-number turbulence. His calculations based on this assumption are in fair agreement with the Uberoi-Corrsin measurements. In view of the lack of accurate information about Lagrangian scales it appears that useful calculations could be made of this quantity from the measurements and theories described in this work.

An estimate of $\mathcal{L}$ can be obtained from *Corrsin*'s [1952*a*] work. The basic estimate, from equation 7 and the definition of $R_{\overline{v\theta}} = \overline{v\theta}/v'\theta'$, is

$$\mathcal{L} = R_{\overline{v\theta}} \frac{\theta'}{(dT/dy)} \tag{11}$$

Using the measured valued of $R_{\overline{v\theta}} = 0.48$ and $\theta'/(dT/dy) = 1.41$ cm (Fig. 13), the computed value of $\mathcal{L}$ is 0.68 cm, or $\mathcal{L}/M = 0.27$. This is in surprising agreement with the data of *Uberoi and Corrsin* [1953].

A secondary estimate for $\mathcal{L}$ can be made from a combination of equations 7 and 8:

$$\mathcal{L} = \frac{6\alpha}{v'} \left(\frac{\theta'}{\lambda_\theta(dT/dy)}\right)^2 \tag{12}$$

Using $\lambda_\theta = 1.1$ cm (Fig. 8) and $v' = 5.3$ cm/sec, which was obtained by measurement, this relation gives $\mathcal{L} = 0.39$ cm, or $\mathcal{L}/M = 0.15$. This calculation is probably incorrect, because it is derived by equating the production and decay of temperature fluctuation. The discussion in section 10 showed that in the measurement region the production and decay were not balanced and there was a growth of temperature fluctuation. If in equation 4 an average rate of growth, as determined by the measurements, is used, the Lagrangian scale can be calculated from the computed value of the production term; the result is $\mathcal{L} = 0.79$ cm, or $\mathcal{L}/M = 0.31$.

Uberoi and Corrsin list the following results for a 1-inch-mesh grid and mean flow of 7.8 m/s: with the line source at $x/M = 43.4$, $\mathcal{L}/M = 0.33$ and the Eulerian integral scale is $L/M = 0.28$; with the line source at $x/M = 86.1$, $\mathcal{L}/M = 0.23$ and $L/M = 0.39$.

Measurements of K_T/K in the heated wake behind a line source in grid-generated turbulence were made by *Uberoi and Corrsin* [1953] and also by *Townsend* [1954]. The asymptotic values of this quantity when interpolated to the grid Reynolds number (UM/ν) used in this experiment were found to be 45 and 38, respectively. These results are considerably different from the value of 17 found in this work. The assumption that the level of heating was not much higher in the other experiments leaves two possible explanations for the difference in turbulent transport. First of all, as has been pointed out above, there is good evidence that the transport mechanism in the present experiment had not reached an asymptotic state. Therefore the appropriate transport value for comparison purposes is probably larger. Second, the mean temperature profile downstream of a line source is Gaussian, and hence at any point in the flow there are higher-order derivatives. It is well known that, when the scale of the transporting mechanism is of the order of the scales associated with the shape of a mean distribution, higher-order derivatives can increase the total transport.

The work of *Dunn and Reid* [1958] yielded results relevant to this discussion. They considered a uniform temperature gradient in a decaying isotropic turbulence of an incompressible fluid. The analysis was carried out by neglecting the triple correlation term in the equation for the double velocity correlation. In addition the spectral distribution of the correlation quantities was taken to be the product of the exponential decay terms (the result of the linearized equations) and the low-wave-number behavior of the general spectral distribution. The low-wave-number behavior is obtained as a consequence of the analytical properties of the spectral functions in the vicinity of zero wave number. This approach is appropriate to the final period of decay of the velocity and temperature fluctuations, because after long decay times (large t) the exponential terms ($\exp -\nu k^2 t$, $\exp -\alpha k^2 t$, where k is wave number) dominate the form of the high-wave-number end of the spectra but the low-wave-number behavior persists.

The analysis shows the following results (using Taylor's hypothesis that $x = Ut$ to get the results in a form that can be compared with the observations in the grid-generated turbulence):

$$\overline{u^2} \sim x^{-5/2}$$

$$\overline{\theta^2} \sim x^{-1/2}$$

$$\overline{v\theta} \sim x^{-3/2}$$

The first result is a well-known one for turbulence in the final period of decay; this problem was studied completely by *Loitsiansky* [1939]. The second result contrasts with the result [*Corrsin*, 1951] for isotropic temperature fluctuation in the final period of decay, which is $\overline{\theta^2} \sim x^{-3/2}$. Evidently the presence of the uniform temperature gradient retards the decay of the temperature fluctuation in the final period. However, the measurements in the initial period of decay showed that the temperature gradient caused a net increase in temperature fluctuation.

Although this report does not include measurements of the streamwise variation of $\overline{\theta v}$, the data of *Townsend* [1954] and also of *Uberoi and Corrsin* [1953] showed that this quantity reached an asymptote where it no longer varied. Similar behavior is expected in grid-generated turbulence with a uniform temperature gradient. The differences between the linearized theory and the experiments emphasizes the different physical situations involved in the two kinds of turbulence, and the difference in their effects on turbulent transport.

13. *Summary and conclusions.* It has been demonstrated that a basic model of turbulent transport can be produced in a wind tunnel with the device of controlled heating at the mixing grid. The temperature fluctuations in this flow are 3 to 4 times higher in rms for temperature gradient heating than for uniform heating with the same input of heating power. Moreover, it has been shown that the 'buoyant' energy drain due to density fluctuations is 1.5 per cent of the energy drain due to viscous dissipation in this experimental flow. Hence, the heating serves as a tagging method with small dynamic alteration of the flow, and the experiment approaches the conceptual model of turbulent transport of a passive attribute (with the possible exception of the effect of the temperature variation of fluid properties).

These results were achieved with a maximum temperature rise of 6°C near the top of the test section and a linear decrease to no rise near the bottom. The temperature fluctuations in this flow were large enough to permit measurement of several interesting quantities by the extension of established hot-wire techniques.

The two-point temperature correlation measurements behaved as might have been predicted on the basis of previous measurements in a uniformly heated flow except that correlations of temperatures at two points along the direction of the temperature gradient had a larger integral scale than the correlations along a normal to the temperature gradient direction. This difference, which is the manifestation of the axisymmetry of the temperature fluctuation field due to the presence of the temperature gradient, is not large on a relative basis or as compared with the computed result of a final period analysis.

The two-point velocity-temperature correlations in the two traverse directions showed a larger difference, but again not as large as the result of the analysis with linearized equations. None of the correlation measurements varied much for the two stream positions $x/M = 102$ and 140 or with and without the additional mixing grid at $x/M = 72$.

The rms temperature fluctuation at a point varied linearly with the mean temperature gradient. This fact was the best experimental verification of the passivity of the heating.

The factors determining the level of mean square fluctuation were (1) production of fluctuation by the interaction of the turbulence and the mean temperature gradient and (2) decay of the fluctuation by the action of molecular transport on the local gradient of the temperature fluctuation. It is expected that the decay term is proportional to $\overline{\theta^2}$, while the production term will reach a constant magnitude quickly. However, the decay term is also inversely proportional to λ_θ^2, which may be proportional to $(x - x_0)$ if it behaves as it does with uniform heating. The consequence of these factors is that $\overline{\theta^2}$ tends toward an asymptotic constant rate of increase because of the growth of the scale of the decaying turbulence (see the Appendix). The measurements show a continual increase of $\overline{\theta^2}$, but they were not sufficiently extensive to prove or disprove the above prediction on a quantitative basis.

The heat transfer correlation coefficient, $\overline{\theta v}/\theta' v'$ was found to be 0.48 from measurements at one point in the flow.

The ratio of 'eddy' to molecular thermal conductivity was calculated to be 17.

The measurements in the initial period of decay of a grid-generated turbulence with uniform temperature gradient show significant differences from the results of the analysis of a uniform temperature gradient in the final period of decay. Although some of the physical processes in this experiment are similar to those involved in the diffusion from a line source experiment, uniform gradient transport is considered a more attractive flow for basic studies because the gradient is maintained constant along the streamwise direction and the flow is homogeneous in planes perpendicular to the flow direction.

Improvement of the experimental arrangement and apparatus and more extensive measurements (especially spectral analyses) along with development of the combined Lagrangian-Eulerian analysis introduced by *Corrsin* [1952*a*] could increase our knowledge of the physical processes involved in turbulent transport.

Appendix

Production and decay of temperature fluctuation in the initial period. Another difference between Corrsin's model and the experimental flow stems from the different 'initial' conditions on the temperature fluctuation. In the model (Fig. 1), $\overline{\theta^2}$ is zero at the imaginary heating plane. The description of the flow in section 4 showed that, until the flow developed to the point where measurements could be taken, $\overline{\theta^2}$ had become relatively high. This difference is complicated by the fact that the behavior of the decay term differs in the two situations. There is sufficient empirical evidence, however, to permit simplifications of the production and decay terms of equation 4 so that the equation yields a simple solution.

It has been demonstrated [*Townsend*, 1954; *Uberoi and Corrsin*, 1953] (section 12) that $\overline{\theta v}$ reaches a constant value in the initial period of decay of grid-generated turbulence. The results of this experiment show that for a first approximation the statistical properties of the temperature fluctuation field can be considered to be isotropic. If in addition it is assumed that λ_θ varies with streamwise position as it does in the uniform heating experiment [*Mills, Kistler, O'Brien, and Corrsin*, 1958], equation 4 can be written

$$\frac{d\overline{\theta^2}}{dx} = -2\,\frac{\overline{\theta v}}{U}\frac{dT}{dy} - \frac{3}{5}\,\frac{\overline{\theta^2}}{(x - x_0)} \tag{13}$$

where $x_0/M \cong 10$ [*Batchelor*, 1956, p. 136]. The solution of this equation is

$$\overline{\theta^2} = -\frac{5}{4}\,\frac{\overline{\theta v}}{U}\frac{dT}{dy}\,M\left(\frac{x - x_0}{M}\right) + C\left(\frac{x - x_0}{M}\right)^{-3/5} \tag{14}$$

The evaluation of the constant C must be made at a point in the flow where the assumptions leading to equation 13 are valid. The description of the flow given in section 4 indicates that such conditions do not begin until $x/M \cong 100$. The measurements show that the second term on the right side of equation 14 is approximately 10 per cent of the other terms and does not get to be 5 per cent until $x/M = 350$. But this region is beyond the initial period of decay, and the decay term no longer has the form used in equation 13. Therefore the experiment with the single grid cannot be expected to display an asymptotic state in which $\overline{\theta^2}$ increases linearly with x.

Calculations were made with equation 14 for the development of temperature fluctuation using the secondary grid at $x/M = 72$. For this calculation the assumption was made that $x_0/M = 82$. The result showed that the decay term was 4 times as great as the production term at $x/M = 102$ and would be 30 per cent of the production term at $x/M = 182$, where it is expected that the new initial period of decay would have its termination.

It is evident that in order to create a flow with linear increase of $\overline{\theta^2}$ it will be necessary to design a system in which the initial $\overline{\theta^2}$ will be close to

$$-\frac{5}{4}\,\frac{\overline{\theta v}}{U}\frac{dT}{dy}\,(x_H - x_0)$$

where x_H is the stream position at which the turbulence has become homogeneous. If this is achieved, $\overline{\theta^2}\,\overline{v^2}$ will be a constant product and hence $R_{v\theta}$ should also remain the same throughout the initial period of decay.

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