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Empirical Data on Turbulence in the Surface Layer of the Atmosphere[1]

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Abstract. The statistical characteristics of turbulence in the atmospheric boundary layer are relatively stable when averaged over a time of the order of 10 to 20 minutes. The large-scale turbulence is related to a set of 'external parameters': $g/\bar{T}$, $q/c_p\rho$, and v_*, where T is the average temperature, q the vertical component of the turbulent heat flux, and v_* the friction velocity. The smaller-scale turbulence is related to a set of external parameters consisting of the rate of dissipation of turbulent energy, the rate of decay of inhomogeneities of the temperature field, and the buoyancy parameter $g/\bar{T}$. For each of these ranges of the turbulence spectrum, two similarity expressions which are functions of these external parameters are determined. Three limiting cases of turbulence in the atmospheric boundary layer are considered: (*a*) neutral stratification where q must be eliminated from the similarity expressions; (*b*) strong instability where v_* is eliminated; (*c*) strong stability where the height above the ground z must be eliminated from the similarity expressions. The time spectra of vertical velocity and of temperature are found and compared with experimental data. Spectra of turbulent stress and heat flux are also considered, and experimental data available about these spectra show that, at a height of 1 meter, the main contribution to the turbulent friction and heat flux is due to wavelengths of horizontal inhomogeneities of the order of some meters or even tens of meters.

Reynolds numbers $Re = \bar{u}z/\nu$ in the surface layer of the atmosphere are usually large (of the order of 10^5–10^8), and turbulence is well developed. One feature of atmospheric turbulence is its dependence upon stratification of the air, characterized by the Richardson number

$$Ri = \frac{g}{\bar{T}} \frac{\partial \bar{T}}{\partial z} \left(\frac{\partial \bar{u}}{\partial z}\right)^{-2} \quad (1)$$

The values of hydrodynamical characteristics in the surface layer, averaged over a time interval of the order of 10–20 minutes, are rather stable statistically. Therefore, we can practically determine the mean velocity and temperature profiles $\bar{u}(z)$ and $\bar{T}(z)$ and turbulent fluctuations u', v', w', and T'.

Turbulence depends upon weather conditions through some 'external parameters.' For components of turbulence outside the dissipation interval of the spectrum, the values of $q/C_p\rho$, v_*, and $g/\bar{T}$ can be taken as a complete set of external parameters (q = vertical turbulent heat flux, v_* = friction velocity). Instead of these parameters we can use their combinations: scales of length, velocity, and temperature

$$L = \frac{-v_*^3}{\kappa \frac{g}{\bar{T}} \frac{q}{C_p\rho}} \qquad V = \frac{V_*}{\kappa};$$

$$T_* = -\frac{1}{\kappa V_*} \frac{q}{C_p\rho} \quad (2)$$

The averaged dimensionless characteristics of turbulence determined by means of scales (2) will be universal functions of dimensionless coordinates (and time) of points of observation. Empirical study of turbulence in the surface layer is directed to the determination of these universal functions.

The mean velocity and temperature profiles can be described by the equations

$$\frac{\partial \bar{u}}{\partial z} = \frac{v_*}{\kappa L} f'\left(\frac{z}{L}\right) \qquad \frac{\partial \bar{T}}{\partial z} = \frac{T_*}{\alpha L} f'\left(\frac{z}{L}\right) \quad (3)$$

where $f'(\zeta)$ = some universal function. According to experimental data we can consider the ratio α of exchange coefficients for heat and

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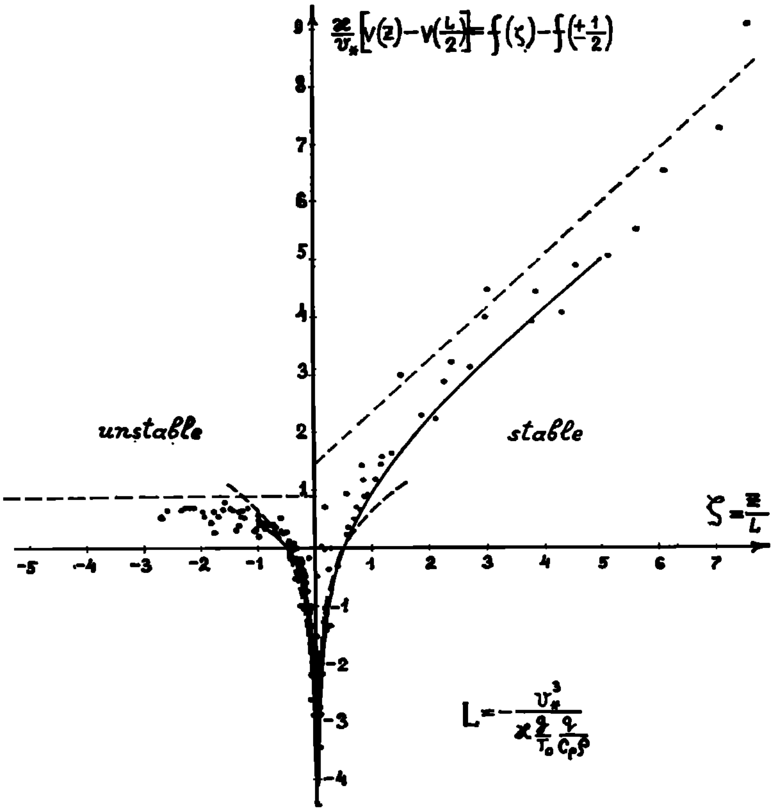
unstable
stable
$\frac{\varkappa}{v_*}\left[v(z)-v\left(\frac{L}{2}\right)\right]=f(\zeta)-f\left(\pm\frac{1}{2}\right)$
$\zeta=\frac{z}{L}$
$L=-\frac{v_*^3}{\varkappa\frac{g}{T_0}\frac{q}{c_p\rho}}$

Fig. 1.

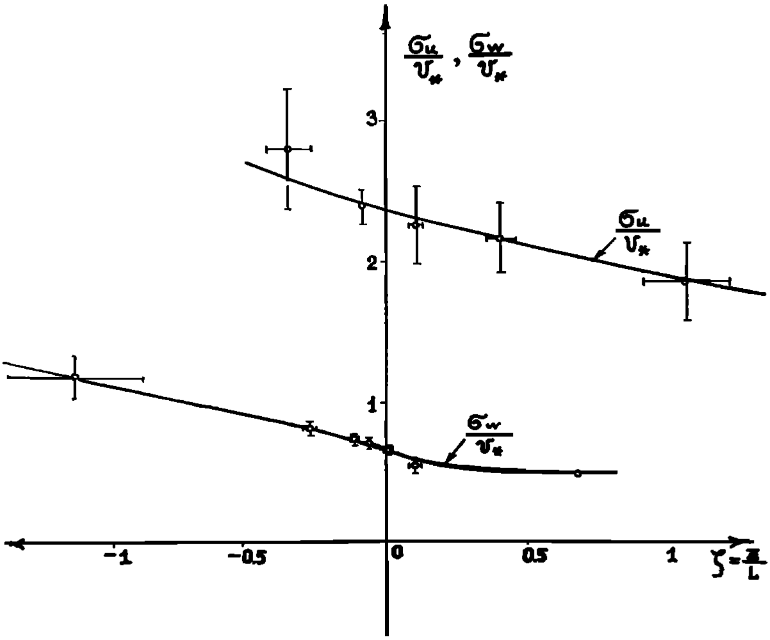
$\frac{\sigma_u}{v_*}$, $\frac{\sigma_w}{v_*}$
$\frac{\sigma_u}{v_*}$
$\frac{\sigma_w}{v_*}$
$\zeta=\frac{z}{L}$

Fig. 2.

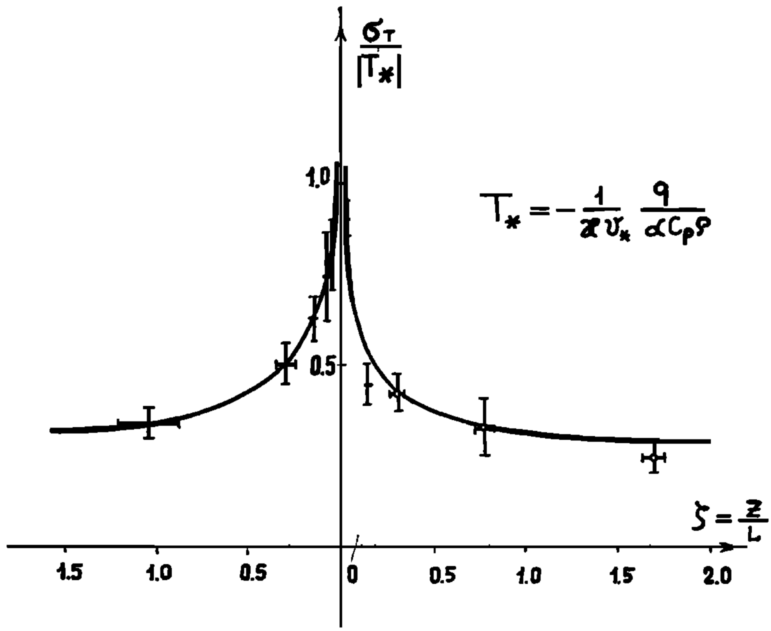

Fig. 3.

momentum to be a constant. In other words, the velocity and temperature profiles have similar form. From (1) and (3) we get

$$Ri = \frac{l}{\alpha f'(z/L)} \tag{4}$$

Therefore $\zeta = z/L$ can be used as a stability parameter as well as Ri.

In the cases of neutral stratification ($q \to 0$ or $z \ll |L|$), strong instability or free convection ($L < 0, z \gg |L|$), and strong stability ($z \gg L > 0$) we can easily determine the asymptotic behavior of universal functions by means of the following principles:

(*a*) In the case of neutral stratification the parameter L must be omitted from the similarity equation of type (3). For instance, we get $f'(\zeta) \approx 1/\zeta$, which corresponds to a well-known logarithmic law for velocity profile.

(*b*) In the case of strong instability the parameter V_* must be omitted from the similarity equations. For instance, $f'(\zeta) \sim -C_1|\zeta|^{-4/3}$.

(*c*) In the case of strong stability the height z must be omitted from the similarity expressions. For instance, $f'(\zeta) \sim c_2$.

Figure 1 shows the shape of universal function

$$\frac{\kappa}{V_*}\left[\bar{u}(z) - \bar{u}\left(\frac{|L|}{2}\right)\right] = f(\zeta) - f(\pm\tfrac{1}{2}) \tag{5}$$

This graph was obtained first by A. S. Monin and A. M. Obukhov in 1953 and later by A. V. Perepelkina, A. S. Gurvitch, and others (by means of independent gradient and fluctuation measurements). The dashed lines are asymptotics, obtained by means of principles (*a*), (*b*), and (*c*). The solid lines are the interpolation curves

$$f(\zeta) = \ln|\zeta| + 0.6\zeta + \text{const}$$

The joint probability distribution for the values of dimensionless fluctuations at a fixed point in space-time depends only upon the parameter $\zeta = z/L$ (or Ri). The first moments of this distribution are equal to zero; all the second moments can be expressed by means of standard deviations σ_u/V_*, σ_v/V_*, σ_w/V_*, and $\sigma_T/|T_*|$ considered as functions of $\zeta = z/L$. By means of these functions we can determine the coefficients of anisotropy σ_v/σ_u, σ_w/σ_u and correlation coefficients r_{uw} and r_{wT} as functions of a stability parameter. The most detailed results about these four functions were obtained by A. S. Gurvitch and L. R. Zvang (Institute of

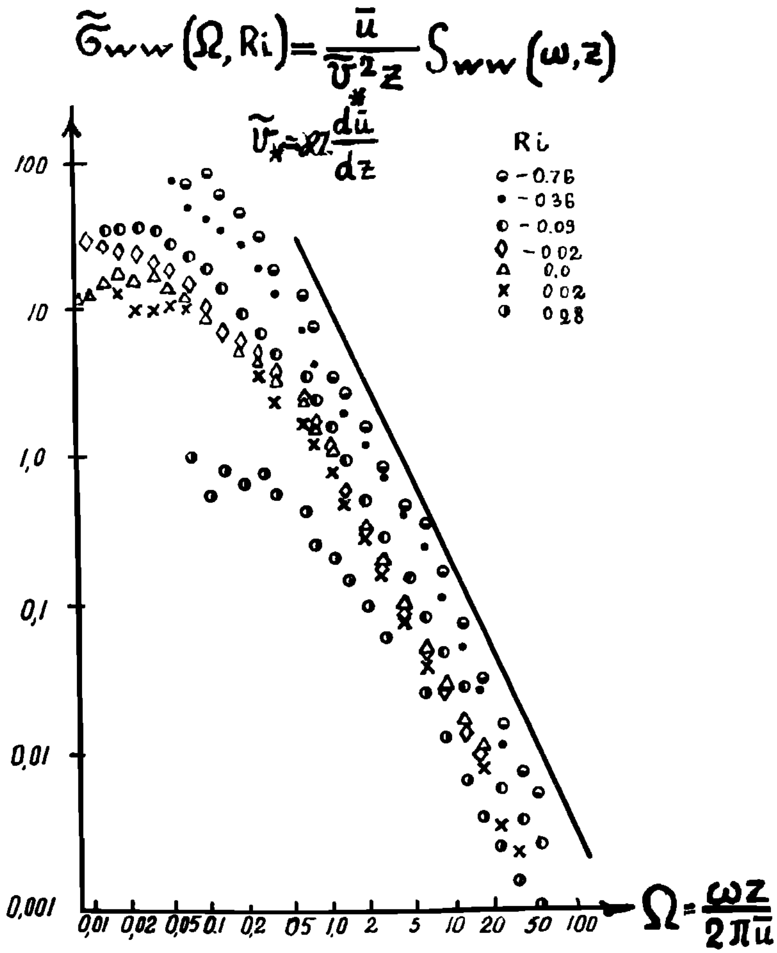

Fig. 4.

Atmospheric Physics, Academy of Sciences, USSR).[2]

Figure 2 shows the functions G_u/V_* and σ_w/V_*. These values decrease as stability increases. Under neutral stratification $\sigma_u/V_* \approx 2.3$, $\sigma_w/V_* \approx 0.7$ (coefficient of anisotropy $\sigma_w/\sigma_u \approx 0.3$; it is slowly increasing with increase of instability).

Figure 3 shows the function $\sigma_T/|T_*|$. Under near-neutral stratification this value exceeds 1 and decreases rapidly with increase of stability and instability.

[2] The following data are looked upon as very preliminary. Some of them were obtained in the first test measurements. To specify and to enlarge these very important data is a significant task of experimental research on atmospheric turbulence.

The available empirical data show that the probability distributions for W' and T' have positive asymmetry.

For components of turbulence from the inertial interval of the spectrum the 'external parameters' are ϵ, the rate of dissipation of turbulent energy; $N = \nu_T(\overline{\nabla T})^2$, the measure of temperature dissipation (the rate of decay of temperature inhomogeneities); and the buoyancy parameter $g/\bar{T}$. The values of ϵ and N depend on z according to the equations

$$\epsilon = \frac{v_*^3}{\kappa z} \varphi_\epsilon\left(\frac{z}{L}\right) \qquad N = \frac{\kappa v_* T_*^2}{\alpha z} \varphi_N\left(\frac{z}{L}\right) \qquad (6)$$

where under near-neutral stratification φ_ϵ, $\varphi_N \rightarrow 1$.

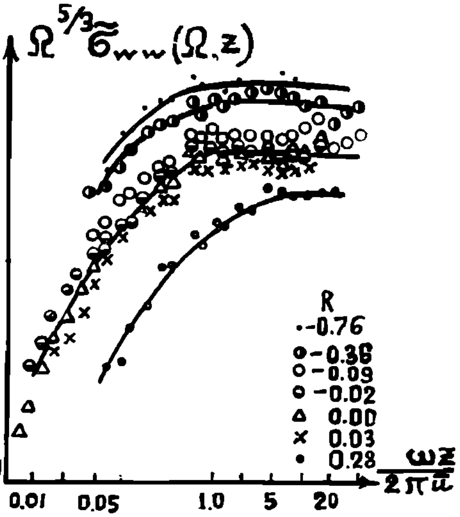

Fig. 5.

The time spectra of W' and T' must have the form

$$S_{ww}(\omega, z) = \frac{V_*^2 z}{\bar{u}} \sigma_{ww}\left(\frac{\omega z}{\bar{u}}, \frac{z}{L}\right)$$
$$S_{TT}(\omega, z) = \frac{T_*^2 z}{\bar{u}} \sigma_{TT}\left(\frac{\omega z}{\bar{u}}, \frac{z}{L}\right) \qquad (7)$$

where ω is frequency, $\omega/\bar{u}$ is wave number, and σ_{ww} and σ_{TT} are some universal functions. The characteristic points of spectra (frequencies of maxima, limits of inertial interval, etc.) must have the form $\omega = (\bar{u}/z)\ \Omega\ (z/L)$. Principles (*a*), (*b*), and (*c*) are valid for dimensionless functions σ and Ω. (In particular, under a strong stability $\Omega \sim \bar{u}/L$; that is, characteristic points of spectra are shifted to high frequencies.) In the inertial interval, using parameters ϵ and N (and neglecting the buoyancy parameter $g/\bar{T}$), it is easy to see that spectra must be proportional to $\omega^{-5/3}$.

Dimensionless time spectra of W' obtained by Gurvitch in the frequency interval $0.01 \leq \omega z/2\pi\bar{u} \leq 100$ are shown in Figure 4. Each curve represents a mean spectrum for corresponding intervals of Richardson numbers from $Ri = -0.76$ to $Ri = +0.28$. Instead of V_* a value

$$\tilde{V}_* = \kappa z(\partial\bar{u}/\partial z) = v_* \cdot \zeta f'(\zeta)$$

was used for the sake of convenience of experimental determination. Figure 4 shows that the values of $\tilde{\sigma}_{ww}$ are decreasing with increase of stability, while the frequency of maximum is shifting to high frequencies. Under neutral stratification the function $\tilde{\sigma}_{ww}$ reaches its maximum near $\omega z/2\pi\bar{u} \approx 0.01$–$0.02$, which corresponds to the wavelength $l = 2\pi\bar{u}/\omega \approx 50$–$100$ meters. Experimental data confirm the validity of the $-5/3$ law for spectra except the low-frequency band.

Figure 5 demonstrates the low-frequency limit of the inertial interval of the spectrum. The values of $\Omega^{5/3}\ \tilde{\sigma}_{ww}$ are plotted on this graph against the dimensionless frequency $\Omega = \omega z/2\pi\bar{u}$. The constant values of $\Omega^{5/3}\ \tilde{\sigma}_{ww}$ correspond to the inertial interval. Figure 5 shows that the low-frequency limit of the inertial interval is shifted to high frequencies by increase of stability. This limit is equal to $\omega = 2.5\ \bar{u}/z$ if $Ri = -0.76$, $\omega = 4.5\ u/z$ if $Ri = 0$, and $\omega = 12\ u/z$ if $Ri = +0.28$. Corresponding scales of length are of the order of height of observation.

Figure 6 demonstrates the spectral energy distribution of $W' =$ fluctuations. The values $\omega S_{ww}/2\pi\sigma_w^2$ are plotted on this graph against $\omega z/2\pi\bar{u}$ for three different values of Ri. The

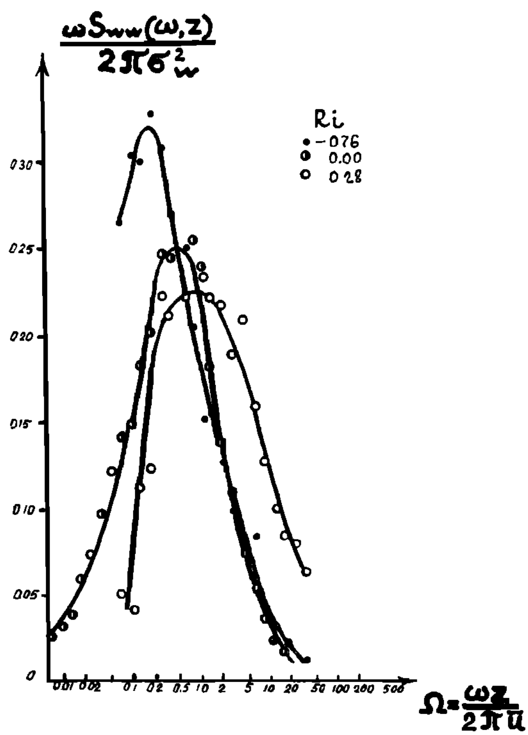

Fig. 6.

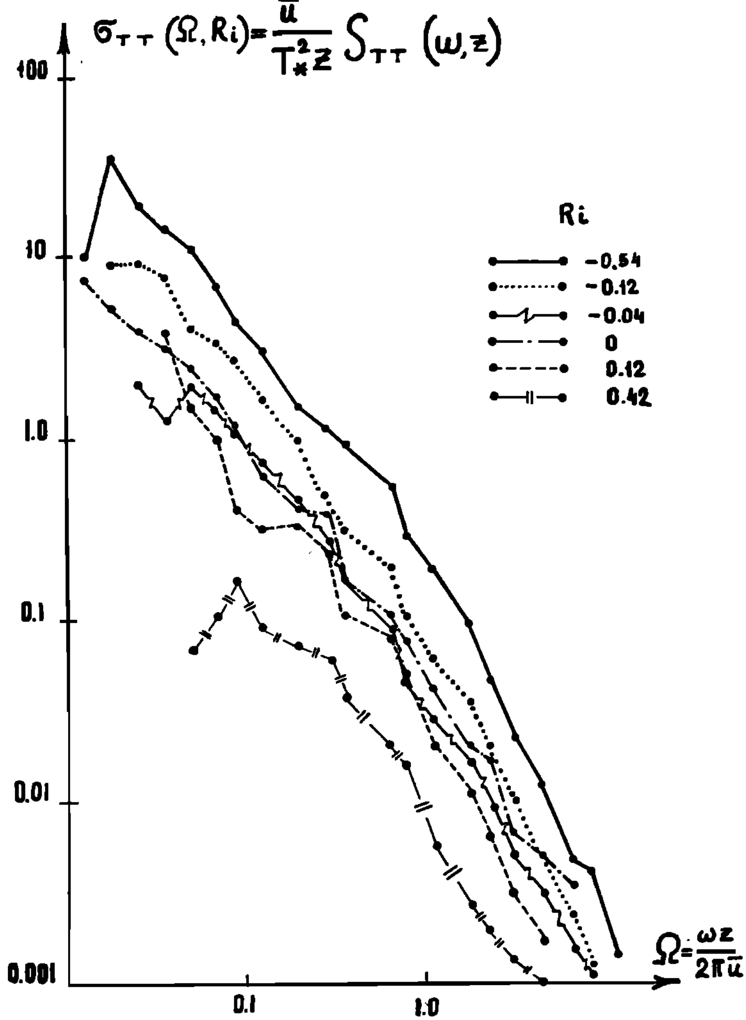
$\sigma_{TT}(\Omega, Ri) = \frac{\bar{u}}{T_*^2 z} S_{TT}(\omega, z)$
100
10
1.0
0.1
0.01
0.001
0.1
1.0
Ri
-0.54
-0.12
-0.04
0
0.12
0.42
$\Omega = \frac{\omega z}{2\pi \bar{u}}$

Fig. 7.

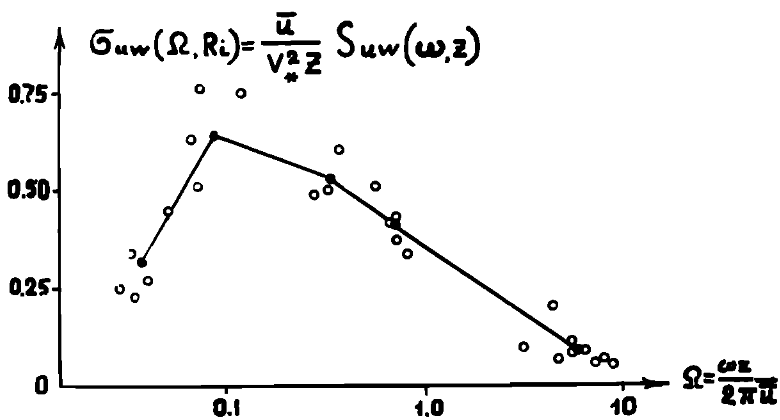
$\sigma_{uw}(\Omega, Ri) = \frac{\bar{u}}{V_*^2 z} S_{uw}(\omega, z)$
0.75
0.50
0.25
0
0.1
1.0
10
$\Omega = \frac{\omega z}{2\pi \bar{u}}$

Fig. 8.

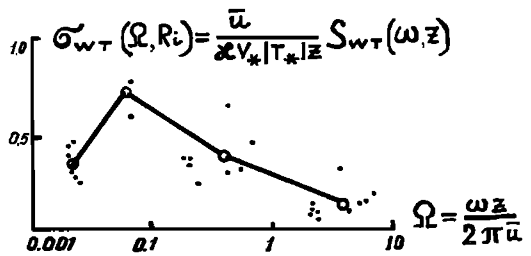

Fig. 9.

curves represent the contribution to the total energy from different frequencies. The maxima of curves correspond to $\omega = 1.5\, u/z$ if $Ri = -0.76$, $\omega = 2\, u/z$ if $Ri = 0$, and $\omega = 5\, u/z$ if $Ri = +0.28$. Figure 6 shows that the inertial interval of the spectrum contains about a third of the total energy.

Dimensionless time spectra of temperature obtained by Zvang are shown in Figure 7. Their features are very similar to those of W' spectra. The data show that the temperature fluctuations under different weather conditions are concentrated in the spectral band $0.03 \leq \omega z/2\pi\bar{u} \leq 3$.

(The dimensionless time spectra of turbulent stress and heat flux must have the form

$$\frac{1}{V_*^2}\,|\overline{u(\delta\omega)\,w(\delta\omega)}| = \frac{\delta\omega \cdot z}{\bar{u}}\,\sigma_{uw}\left(\frac{\omega z}{\bar{u}}, \frac{z}{L}\right);$$

$$\frac{1}{\kappa V_*\,|T_*|}\,|\overline{w(\delta\omega)\,T(\delta\omega)}| = \delta\omega \cdot z/\bar{u}\,\sigma_{wT}(\omega z/\bar{u},\, z/L) \qquad (8)$$

where $\delta\omega$ is an infinitesimal frequency interval, σ_{uw} and σ_{wT} are some universal functions.) The examples of measurements of σ_{uw} and σ_{wT} (A. S. Gurvitch and L. R. Zvang; $z = 1$ meter; weak instability) are shown in Figures 8 and 9. The spectra σ_{uw} and σ_{wT} are somewhat similar. Figures 8 and 9 show that the main contribution to the turbulent friction and heat flux at a height of 1 meter is made by the frequencies $\omega z/2\pi\bar{u} \sim 0.1$–$1.0$ which correspond to wavelengths of horizontal inhomogeneities of the order of some meters.