Upper Atmospheric Turbulence Determined by Means of Rockets[1]

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Abstract. The motion field of the upper atmosphere was studied with the aid of sodium trails brought to the 100-km level by means of a Véronique rocket. Below 102 km the trail shows globular distortions in the form of elements having an average diameter of 0.5 km; above that height the trail is completely smooth, though highly curved, showing no small irregularities. The turbulence state of the upper atmosphere was studied by investigating diffusion of the sodium atoms. Above 102 km the broadening of the trail can be completely accounted for by molecular diffusion; below that height the observed rate of increase of the globule diameters can be explained only by assuming a combination of molecular and turbulent diffusion. This information and the increasing chemical differentiation above 102 km strongly indicate that the 100-km level marks the difference between the turbulent and the nonturbulent parts of the upper atmosphere. The absence of turbulence in the upper atmosphere is bound to be due to the increasing relative importance of the friction forces there. The Reynolds number rapidly decreases with increasing atmospheric height. The absence or occurrence of turbulence cannot be due to shear effects: the Richardson number is too great in the region studied by us, which shows that shear turbulence is not to be expected in this part of the earth's atmosphere. Since the energy fed into the turbulence field is rather small, turbulence cannot be fully developed; in fact, it is predicted that the turbulent element can only have diameters smaller than about 2 km, which is not in disagreement with the observed element diameter of 0.5 km. The smallest visible wavelength of the motion field is of the order 50 to 100 meters, in agreement with ionospheric investigations (20 to 60 meters) and theoretical predictions (60 meters).

1. *Observations.* The results presented in this investigation are based mainly on the study of a trail of sodium vapor brought by a Véronique rocket into the earth's upper atmosphere between heights of 80 and 120 km. The firing took place on March 10, 1959, at 1840 UT in the Sahara about half an hour after sunset. The sky was already dark, but in the upper atmosphere the sodium trail was illuminated by sunlight and was thus visible as a yellow trail against the dark sky. The observations show that the trail is very rapidly deformed by the strong windshear prevailing in the upper part of the upper atmosphere. An essential feature, however, seems to be that the part of the trail above a certain level (102 km) is smooth, though highly curved, whereas the lower part is more turbulent. The trail consists of elements having an average diameter of 0.5 km. The division level between these two parts of the trail is rather sharp, not wider than about 1 km. This 'turbulence' is a real atmospheric effect; it cannot be due to interaction of the rocket with the surrounding air, since it is observed both when the rocket is ascending and when it is descending and since other observers with other instruments have measured similar affects; they also found the transition to occur at the same height level [*Groves and others*, 1960].

In this paper we examine whether the irregular aspect of the trail below 102 km is really due to turbulence, turbulence being defined in the aerodynamical sense: as a motion field so irregular that it can be described only by statistical methods, energy being fed into it in the region of long wavelengths and dissipated by viscosity in the region of smaller wavelengths. One of the most important ways to investigate

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this question is the study of the diffusion of the gas molecules in the region concerned.

2. *Turbulent and ambipolar diffusion.* It was shown by *Blamont* [1960] that in the region *above* 102 km we are dealing with molecular diffusion only, not with turbulent diffusion. The theory of ambipolar diffusion shows that the distribution of the sodium atoms as a function of the distance r to the center of the trail should be Gaussian: exp $(-r^2/L^2)$. A plot of L^2 as a function of the time t since the origin of the trail should be a straight line and should permit one to derive the diffusion coefficient. Such measurements made for altitudes of 104 and 113 km, hence in the smooth part of the trail, yielded perfectly linear relations between L^2 and t, and diffusion coefficients in agreement with the theory of molecular diffusion (Fig. 1). Hence, turbulent diffusion is negligible in the region above 102 km.

The same does not apply to the observations *below* 102 km. In that region the speed of growth of the globules was examined on a series of photographs of a trail, taken on March 2, 1960, with a Newtonian telescope in fairly rapid succession and with rather short exposure times. On the photographs, which refer to the descending part of the trail, the diameters of single globules were measured. One minute after the ejection of the trail the average diameter of the globules was 90 meters; it increased to 500 meters in 120 seconds; hence the speed of expansion was 3.3 meters per second.

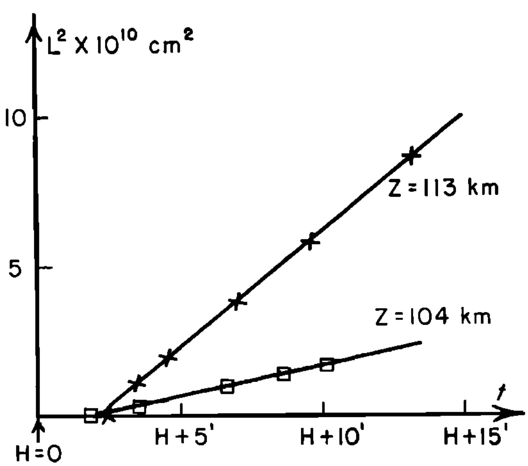

Fig. 1. Diffusion as a function of time.

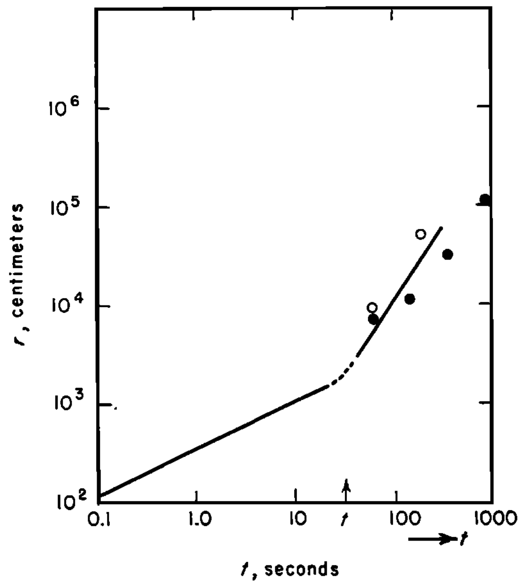

Fig. 2. Molecular and eddy diffusion (after *Greenhow* [1959]). Dots: average diameters of meteor trails, as measured by Greenhow. Open circles: diameters of sodium globules, measured in the trail of March 2, 1960. The lines show the suggested variation of the globular diameter r as a function of the time t.

$t < t_1$: molecular diffusion : $r^2 = 4\ Dt$.

$t_1 < t < t_2$: transition region : $r^2 = 4/3\ wt^3$.

$t > t_2$: pure eddy diffusion : $r^2 = 4\ v_2 l_2 t$.

From the graph we derive $t_1 \approx 30$ sec; $t_2 > 15$ min; $w \approx 70$ erg g^{-1} sec^{-1}.

These results can be compared directly with those derived from the diffusion of meteor trails observed by radar technique. Figure 2 compares our measured globular diameters with diameters of meteor trails observed at a slightly lower height level [*Greenhow*, 1959]. The agreement between these different kinds of results is fairly satisfactory.

It appears that these observations can be described rather well by the relation

$$\log r = \log t + \text{constant}$$

whereas molecular diffusion theory would require

$$\log r^2 = \log t + \text{constant}$$

therefore the theory of ambipolar diffusion does not apply to these observations.

The theory of turbulent diffusion in the earth's upper atmosphere has been stated by *Booker* [1956] and by *Booker and Cohen*

[1956]. An element of gas first expands according to molecular diffusion, with a diffusion constant $D = 4$ meter^2 sec^{-1}, at the 90-km level until a time t_1 after the ejection of the cloud. Here t_1 is the time constant of the smallest elements of the turbulence field. We have $t_1 = l_1 v_1^{-1}$, where l_1 is the scale of the smallest elements and v_1 the average turbulent velocities of these elements.

If t_2 is the time constant of the large elements of the field, then for a time $t > t_2$ after the ejection of the trail the rate of dispersal of the trail is given by the coefficient of eddy diffusion

$$D_e = v_2 l_2$$

where v_2 and l_2 are the velocity and scale of the large-scale turbulence.

It can be shown that for the time interval between t_2 and t_1 the coefficient of diffusion is wt^2, where w is the rate of supply of energy to the large eddies of the turbulence field.

The consequent expansion velocity of the trail is given by

$$\begin{aligned} r^2 &= 4Dt && \text{for} \quad t < t_1 \\ r^2 &= \tfrac{4}{3}wt^3 && \text{for} \quad t_1 < t < t_2 \\ r^2 &= 4v_2 l_2 t && \text{for} \quad t > t_2 \end{aligned}$$

Comparison of this theory with the observations shows, as has already been noticed by *Greenhow* [1959], that molecular diffusion remains the main broadening mechanism until about 1 minute after the formation of the trail. This behavior and the subsequent expansion show that w, the rate of supply of energy to the field, is about 70 erg g^{-1} sec^{-1}. This value is smaller (by a factor 3.10^3) than the value initially predicted by *Booker* [1956]. The value of t_1 is about 30 sec, which, assuming $l_1 = 60$ meters (see section 7), yields $v_1 = 2$ meters/sec, a not unreasonable value.

Remark. It will be noticed that the relation between r and t found by us in the transition region ($t > t_2$) does not fulfill exactly the law predicted by Booker. The slope found by Greenhow and us is slightly less. The reason for this small discrepancy may be found in the fact that the motion field of the upper atmosphere is not a Kolmogorov field, which has indeed been implicitly assumed by Booker in developing his theory (see, e.g., *Booker* [1956], equation 8, p. 680; see also section 3).

3. *Statistics of the motion field in the upper atmosphere.* To give an idea of the motion field in the earth's upper atmosphere we have selected two photographic observations obtained from the station Barga at 18h 42m 0s and 18h 45m 30s, where the rocket was ascending and descending, respectively. A slightly twisted descending trail had been formed but it was still not deviating much from linearity, which allowed us to assume that the displacements were proportional to the local wind velocities. The displacement of the trail, as compared with the path of the rocket, was measured for 90 points in both the ascending and the descending parts of the trail. The measured part was situated between altitudes of 102 and 86 km. The height difference between two consecutive points was thus 0.18 km.

For these 90 points we have measured the average velocity difference

$$\{\Delta v(z)\}^2 \equiv \langle [v(x) - v(x + z)]^2 \rangle$$

where z has been given values between 0.18 and 9 km, and where mean values have been taken over all possible x values along the trail. According to Kolmogoroff's law we should expect

$$\{\Delta v(z)\}^2 \sim z^{2/3}$$

but only for very large values of Re and for small values of z.

Instead, we obtained a much higher value of the exponent (Fig. 3), namely 1.45 for the ascending part and 1.60 for the descending part. The mean is 1.51. To examine this result further we applied a similar treatment to observations obtained at another Véronique firing, that of March 2, 1960, at which photographs showing much smaller details were obtained by J. Texereau with the aid of a Newton telescope having an objective diameter of 250 mm and a focal length of 1255.5 mm. The film was Kodak Tri X; the exposures were taken through a Wratten filter 23A. Two photographs taken at 18h 35m 0s and 18h 35m 06s, or 7 minutes and 7 minutes 6 seconds after the launching, with exposure times of 1 and 2 seconds, respectively, revealed sharp images of sections of the descending part of the trail.

A part of the trail has been subjected to a spectrum analysis, as described above, yielding the result that here too Kolmogoroff's law is not satisfied; we find

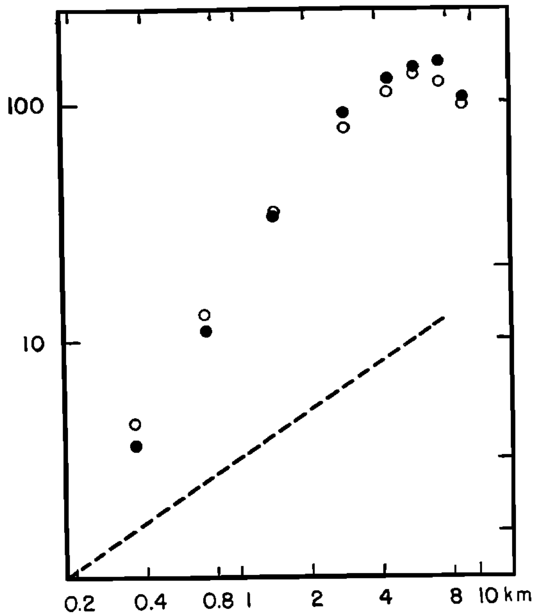

Fig. 3. The spectrum of the wind motions (presumably *not* of turbulence): log $\{\Delta v(z)\}^2$ against log z. Dashed is the gradient of the Kolmogoroff spectrum (= 2/3). Trail of March 10, 1959. Open circles, ascending part; dots, descending part.

$$\{\Delta v(z)\}^2 \sim z^{1.3}$$

the exponent being the average of those derived for the two photographs (1.4 and 1.1). This result essentially confirms that derived from the firing of March 10, 1959.

In the spectrum of March 10, 1959, a clear maximum is visible at a vertical distance $z \approx$ 6 km, showing that the wind motions in the upper atmosphere have a correlation distance of that order.

The same conclusion was also reached by an investigation of radar reflections of long-lived meteor trails by *Greenhow* [1959] and *Greenhow and Neufeld* [1959], who studied the correlation between the wind velocities at two points at different distances and found that the motion field is anisotropic. The vertical correlation distance is 7 km; the horizontal correlation distance is of the order of 150 km.

We therefore conclude that the assumption of taking the correlation distance of the atmosphere equal to the scale height finds a justification in the present study of the Véronique sodium trail of March 10, 1959, and in the earlier studies of meteor trails.

4. *The Reynolds and Richardson numbers.* The next point is to account for the change in character of the motion field at the 102-km level. That turbulence occurs below 102 km might be due to the decrease of the kinematic viscosity below that level (hence an increase of the Reynolds number above a certain critical value), but it might also be due to wind shear (leading to a Richardson number exceeding a certain critical value).

In normal hydrodynamical flow experiments the Reynolds number is defined as the ratio between the forces of inertia and friction, vl/ν, where v is the velocity of the gas with respect to the walls of the experiment tube, l is the diameter of the tube, and ν is the kinematic viscosity. In the present case, where we are dealing with a free atmosphere, neither v nor l can be defined in the classical way. Therefore we must introduce a *new definition* of the Reynolds number, which in a sense has nothing to do with the classical definition and which could, if necessary, be given another name. Let us call it the Reynolds number for a free atmosphere. In this new definition, we replace l by H, H being the length corresponding to the maximum of the turbulence spectrum or the vertical correlation distance. This vertical characteristic length of the motion field is of the same order as the scale height of the atmosphere (essentially the same conclusion as was reached by *Greenhow and Neufeld* [1959]. For the velocity v we should take the average absolute value of the relative velocity vector

$$v \equiv \langle|\mathbf{v} - \bar{\mathbf{v}}|\rangle$$

where we should further take into consideration that $\bar{\mathbf{v}}$ is mostly close to zero, and that the mean should be a 'running' one, taken over a height region of, say, 5 to 10 km.

Having thus defined the Reynolds number for the free atmosphere, we are able to compute its value for different heights.

The numerical computations were made for two modern model atmospheres published almost simultaneously: the ARDC model of *Minzer, Champion, and Pond* [1959] and that of *Nicolet* [1959].

The ARDC model 1959 gives the viscosity only up to a height of 90 km; we have computed this function above that height with the well-known but rather rough expression

$$\nu = \tfrac{1}{3}\bar{c}l_f \tag{1}$$

where c is the mean molecular velocity and l_f is the mean free path of the gas particles. Errors of a factor 2 or even more are perhaps possible, owing to the approximate nature of equation 1 and to uncertainties in the model atmosphere used. In Nicolet's model atmosphere ν is given in tabular form for the whole region of interest to us.

Values for the wind velocities as a function of height were derived from the measurements made at the same rocket firing (see Fig. 4), whereas a value for lower heights (85 km) has been taken from radar observations of meteors [*Greenhow and Neufeld*, 1959]. Open circles in Figure 4 represent uncertain values; the solid curve has been drawn by hand.

The resulting values of log Re are shown in Figure 5. This figure shows that at the height of transition (102 km) $Re \approx 2 \times 10^3$ with an estimated uncertainty of a factor 2. In fact the value 2×10^3 is reached at a height of 97 km in the ARDC model and at 108 km in Nicolet's model. At the observed transition height (102 km) $Re = 1200$ for the ARDC model and 3000 for Nicolet's model. The present discussion shows that apparently in the upper atmosphere well-developed small-scale turbulence is possible only for $Re > 2000$, if in the computation of Re l_f is put equal to the local scale height H of the atmosphere. The differences between the results found for the two models clearly reflect the uncertainties still existing in upper atmospheric models.

The results of this section show that indeed the increase of kinematic viscosity with height leads to a strong decrease of the Reynolds number with increasing height, and this in fact could be the cause of the inhibition of turbulence above a certain level. Interestingly, the absence of turbulence above 100 km was predicted earlier by *Stewart* [1959] and by *de Jager* [1952].

To investigate the next point, whether shear turbulence can be expected in this part of the atmosphere, the generalized Richardson number has been computed:

$$Ri^* = \left(\frac{l_h}{l_m}\right)^2 \frac{g}{T}\left(\frac{dT}{dh} + \Gamma\right)\left(\frac{d\bar{v}}{dh}\right)^{-2} \tag{2}$$

In this expression l_h is the diameter of the turbulence elements, l_m the momentum mixing length, g the acceleration of gravity, t the temperature, and Γ is not critical for the following part of this paper. Essentially the Richardson number is proportional to the quotient of the gain of potential energy of a gas volume brought

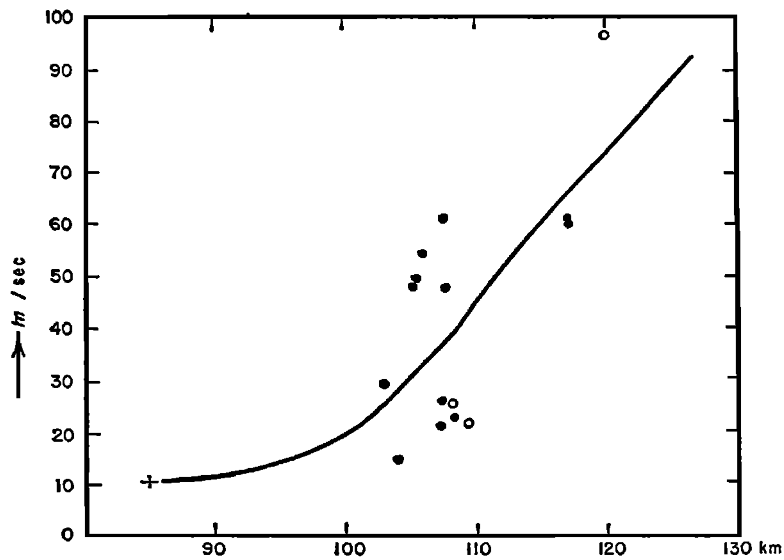

Fig. 4. Wind velocities in the 100-km region, derived from measurements of the trail of March 10, 1959, kindly communicated by Miss Baguette. Open circles represent uncertain values. The plot at 85 km is derived from meteor measurements [*Greenhow*, 1959].

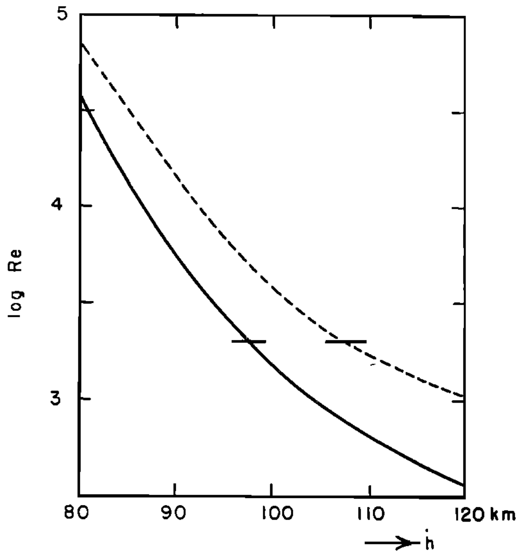

Fig. 5. The computed Reynolds number in the high atmosphere. Solid curve: results for the ARDC model. Dashed curve: for Nicolet model. The critical Reynolds number ($=2 \times 10^3$) is reached at 97 km and 111 km for the two model atmospheres.

upward, and the kinetic energy to which this displacement is due. l_h is the distance over which the volume of gas can be displaced as the result of the kinetic energy, derived from the shear velocity field of the gas.

With the usual assumption $l_h = l_m$ the Richardson number returns to its usual definition and can be computed through the atmosphere. Here we have assumed $\Gamma = 9.9 \times 10^{-5}$ deg K cm^{-1}, a value valid near the ground level of a dry atmosphere. At the 100-km level it may be somewhat less, but its value is not critical.

The Richardson number thus simplified has been computed for the whole atmosphere between 80 and 120 km; its value is found to be everywhere much greater than unity, which shows that the shear velocities of the motion field yield an insufficient energy to produce turbulence in that region of the atmosphere. Furthermore, nothing particular happens with the Richardson number in the region near 102 km. This also proves that viscosity rather than shear effects determines the existence or non-existence of turbulence in the upper atmosphere. This conclusion is in agreement with one reached earlier by *Sheppard* [1959], who stated that shear turbulence is not to be expected in the mesosphere and thermosphere.

However, it can be shown that, in the part of the atmosphere where according to the Reynolds criterion turbulence can exist, the motion field can give rise only to rather small globules and not to a well-developed system of turbulent motions with elements having an average diameter of the order of the scale height of the atmosphere. Turning back to the generalized form of the equation for the Richardson number (equation 1), we might ask for the ratio l_H/l_m for which the generalized Richardson number Ri^* is smaller than unity. It appears that throughout the part of the atmosphere studied by us this is the case for $l_h/l_m \approx 0.3$. This means that the elements of turbulence in the part of the atmosphere where turbulence can exist will have a limited diameter. Assuming $l_m = 6$ km, we obtain $l_h = 2$ km. Hence, we find that the maximum possible globule diameter is of the order of 2 km. This is not in contradiction to the observation that the average diameter is about 0.5 km. Mostly the globules fade away when growing, so that the evolution of globules of 1 km or more cannot be studied. It would be interesting to emit a number of small, discrete, very dense globules and measure their expansion rate, especially for diameters near 2 km.

We conclude that, even in the regions of the atmosphere where turbulence can develop (according to the Reynolds criterion this can happen only below about 100 km), the shear turbulent energy is small, so that it is possible to develop only rather small turbulent elements, with maximum diameters of the order of 2 km; larger elements cannot develop. These maximum values of l_h are in reasonable agreement with the observed average diameters, about 0.5 km.

Another quantity of interest is the smallest wavelength of the turbulence field. It can be found only with the aid of photographs taken with very high resolving power. Such pictures have been obtained of the trail of March 2, 1960, using the Newtonian telescope.

On these photographs, the trails do not show motions with wavelengths shorter than about 2′, corresponding, at the distance of the trail (120 km), to a length of 70 meters. Hence we conclude that the smallest visible wavelengths in the turbulent part of the cloud are of the order of 50 to 100 meters. This value is in good agreement with that found from the scattering of radio waves.

From the theoretical side this value can be perfectly understood. The theory of isotropic turbulence shows that the wave number k_d where the viscous effects become very strong is given by (cf. *Hinze* [1959], p. 184):

$$k_d = (w/\nu^3)^{1/4}$$

where w is the rate of energy supply to the motion field and ν is the kinematic viscosity. In section 3 it was shown that at the 90-km level, hence in the turbulent region, $w \approx 70$ erg g^{-1} sec^{-1}, while ν is of the order 4×10^4. Hence the smallest wavelength $\lambda_d = 2\pi/k_d$ is of the order of 60 meters, which closely resembles the smallest wavelengths observed.

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