Recent Developments in the Theory of Wave Generation by Wind[1]

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Abstract. An account is given of the interplay between the various mechanisms responsible for the generation of waves. When the phase speed c of the waves is much less than the wind speed U, the resonance mechanism associated with the atmospheric turbulence pressure fluctuations provides a trigger for the instability (or sheltering) mechanism, and the rate of growth of the spectral density at a given frequency increases from a linear to an exponential function of time. The consequences of this transition from one mode to another are discussed in the light of the available experimental evidence, and some predictions are made about the directional distribution of the low-frequency components of the spectrum. For frequencies for which $c \approx U$, the energy is gained from the wind largely by the mechanism of resonance. The directional properties of these components are characteristically different: they display maxima at angles $\cos^{-1} c/U$ to the wind direction. The predictions of the theory are compared with observations made in Britain and in the United States. If the wind continues to blow, the high-frequency components rapidly attain a state of saturation, determined by the stability of the air-water interface. The form of the spectra can be determined simply on dimensional grounds, and the predictions agree well with a number of observations under widely differing conditions.

1. *Introduction.* The aim of this paper is to present some of the physical ideas that have emerged in the last few years concerning the problem of the generation of ocean waves. This problem is complex, and a number of different mechanisms are involved whose interplay is responsible for the statistical properties observed in the random wave field of the oceans. We seek here to describe these mechanisms and their interrelations as far as they are known, using mainly a descriptive physical approach. Most of the detailed analyses have been published elsewhere, and references to them will be given, but this paper will have succeeded in its aim if the results of the analysis are made to seem plausible and relevant to some part of this intricate process.

2. *The mechanisms of wave generation.* In generating a wave field on the surface of an ocean initially at rest, two simple mechanisms seem important. The first is the resonance between the surface wave modes and the convected surface pressure fluctuations associated with the turbulent wind blowing over the water. The second is the gradual instability caused by the air flow over a surface already disturbed.

The existence of the resonance mechanism as an important factor in the generation of ocean waves was advanced by *Phillips* [1957]. The wind is, in nature, invariably turbulent, and the atmospheric turbulence results in a random pressure distribution on the water surface that is convected by the mean flow and at the same time is continually evolving as the turbulent eddies above the water interact, grow, and decay. The response of the water surface depends on the magnitude of these pressure fluctuations and also on the time scale of the fluctuations of a given wave number, or, roughly, the time over which the pressure and wave components remain coherent as they both move along. This time scale might be expected to be a maximum when the convection velocity U of the pressure field is just equal to the phase velocity c of free surface waves of the same wave number. Under these conditions, the forcing pressure disturbance remains 'in step' with the forced wave motion for the longest time, until the interaction among the turbulent eddies modifies the surface

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pressure distribution. For wave components traveling at an angle α to the wind, it is the component of the wind speed U in the direction of wave propagation that is relevant, so that the condition for resonance is

$$c = U \cos \alpha \tag{1}$$

and, if this is satisfied, the energy transfer from wind to waves is operating by this mechanism under optimum conditions. If this were the only way that the waves gained energy, we might expect directional maxima for a particular frequency at angles $\pm\cos^{-1}(c/U)$ to the wind.

Within a few months of the publication of this paper, an entirely different investigation by Miles [1957] showed that, if an air stream with a prescribed velocity profile blows over a water surface, any small disturbance is dynamically unstable but the amplification rate is very slow. This result is in some ways reminiscent of some early considerations of *Jeffreys* [1924], in which he suggested that the magnitude of the surface pressure variation induced by wind blowing over a wavy surface (or rather the component in phase with the normal velocity, which does work on the surface) could be expressed in terms of a 'sheltering coefficient.' Although the details of the air-flow patterns visualized differ in the two cases (Miles' analysis is a small perturbation theory and does not envisage actual flow separation at the wave crests), the instability theory is equivalent to a calculation of the sheltering coefficient for small wave slopes. The amplification factor in Miles' theory depends on the shape of the velocity profile near the height at which the wind velocity is equal to the phase velocity c of the waves. At this height, Miles finds a critical layer where the energy is extracted from the wind and transferred downwards to the water. The rate of energy input to the waves is proportional to the wind profile curvature at this critical layer. For relatively slowly moving waves, this critical layer is close to the surface where the profile curvature is large, so that the rate of energy input to the waves is correspondingly large. However, when $c \approx U$, (where U is the wind speed as usually measured several meters above the surface, or about 9 times the friction velocity), the critical layer is now several meters above the surface where the curvature of the profile is small, so that the rate of energy transfer is correspondingly small. It seems then, that the instability is a very effective mechanism in the growth of the shorter wavelengths of the wave field but less so for the longer, faster waves.

As in all linearized instability theories, the rate of growth is proportional to the magnitude of the disturbance already present, so that the wave height is an exponential function of time. In other words, if the initial wave field is very small, so is the rate of growth, but it increases more rapidly as time goes on. On the other hand, the resonance mechanism is independent of the waves already existing, and provides a linear rate of growth of wave energy. On a priori grounds, then, it seems likely that, if the wind begins to blow over an initially calm sea, then, for the components for which $c \ll U$, the initial rate of growth will be linear (from resonance with the atmospheric pressure fluctuations) until the waves become large enough for the instability to take hold. After this the rate of growth will rapidly become larger, so that, in this sense, the resonance mechanism may provide a trigger for setting off the instability.

However, when $c \approx U$, that is, for waves traveling at about the same speed as the wind, the instability is never very effective, since the amplification rate is so slow, whereas the resonance mechanism is operating at its best. Any wave components observed at these frequencies ($\omega = g/c \approx g/U$) are therefore presumably the result of the latter mechanism.

Recently, *Miles* [1960] has given the combined effect analytical form, and in a simple analysis incorporating both mechanisms has verified the above expectations. The time history of a typical Fourier component of the wave spectrum, as given by this analysis, shows a linear rate of growth for small times followed by a more rapid growth at large t. An interesting result from the analysis is that, if the waves start from rest, then, for given frequency and wind properties, the time elapsed until the transition between resonant and unstable growth is independent of the magnitude of the atmospheric pressure fluctuations. Of course the *amplitude* of the wave motion is proportional to the pressure fluctuations, but the transition time is a function only of the shape of the velocity profile and $U \cos \alpha/c$. This result is perhaps a little surprising at first but can be viewed in the following light. If we increase the magnitude of the pressure fluctuations, keeping all else the same, the waves grow more rapidly by the resonance mechanism. However, this provides a better start for the instability, so that its effects are enhanced at the same time. It therefore does not seem implausible that the interval of time after the start of the motion until the instability catches up should be independent of the magnitude of the pressure fluctuations, since variations in these do not favor either mechanism over the other.

The importance of this result is that it enables us to calculate the transition duration as a function of c/U for $\alpha = 0$ in terms of the properties of the wind profile and without reference to the atmospheric pressure fluctuations, about which our information is slight. The calculations are given elsewhere [*Phillips*, 1961], but the results are shown in Figure 1. Suppose we choose a particular frequency of the wave field (a particular value of c/U). At first the wave growth is under the action of the resonance mechanism, but as time goes on the transition duration is reached and the growth becomes exponential. We will refer to the experimental points later. Two aspects of this curve are worth noting. The first is that the transition duration t_t depends very sensitively on c/U; when c/U changes from 0.2 to 0.8, t_t changes by several decades. When $c \approx U$, the transition duration is very large indeed, greater, in fact, than values usually found in the ocean. The second point is that the curve is calculated assuming initial conditions of rest. If the atmospheric pressure fluctuations are small, as present indications suggest [*Longuet-Higgins, Cartwright, and Smith,* 1961], the initial rate of growth is small, so that a wave disturbance already existing when the wind begins to blow may *decrease* the time required for transition, perhaps considerably. In practice, then, the transition duration would be expected to be not greater than as shown in Figure 1.

This is approximately where this part of the theory stands at the moment. Clearly, there are a number of important questions outstanding. *Miles'* [1960] theory assumes that the pressures experienced by the surface are simply the sum of those associated with the atmospheric turbulence and the air flow over the wavy surface. How true is this? What interaction effect is there in the air stream? Is the velocity profile over water really logarithmic, as Miles assumes? (See recent comments by *Stewart*

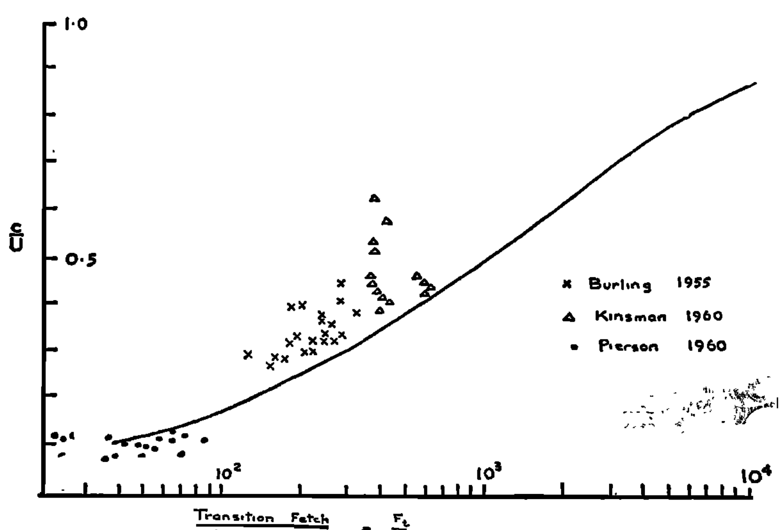

Fig. 1. The transition curve for the change from resonant wave growth to unstable wave growth, in a fetch-limited situation (waves developing from a shore line). In the duration-limited case, with an infinite sea initially at rest, the abscissa represents $(4\pi)^{-1}$ $(\omega_t\ t_t)$ or half the number of wave periods over which the wind has been blowing. The experimental points all refer to fetch-limited cases. Pierson's values are from an unpublished communication on wind-generated waves in a wind-tunnel tank.

[1961].) Another point is that, when the transition duration is attained, the wave amplitudes may already be so large that the amplification factor calculated assuming infinitesimal amplitude disturbances may be in serious error. There may even be boundary-layer separation and an air flow rather closer to that envisaged by Jeffreys. In spite of these questions, however, we will see in section 4 that a number of predictions of the theory as it has been developed so far are borne out experimentally, so that it is likely that the simplifying assumptions have not lost too much of the complex physical reality.

3. *The equilibrium range.* The wave growth as described in the previous section clearly cannot proceed indefinitely. Sooner or later, if the wind continues to blow, the waves may attain a state of saturation over at least part of the spectrum.

Now, what physical process limits the wave growth? Fairly evidently, it is the formation of whitecaps, or white horses. In more analytical terms, it is the gravitational stability of the surface: the particle accelerations cannot exceed g, the acceleration due to gravity. If there is a local excess of energy, the waves break forming whitecaps, and the energy is lost from the wave system to turbulence in the water.

This process is discussed by *Phillips* [1958]. In the limiting equilibrium (or saturated) state, the form of the wave spectrum must be determined by the physical parameters involved in the gravitational stability of the free surface, that is, g and the frequency ω or wave number k. Thus, on dimensional grounds the frequency spectrum

$$\Phi(\omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} \overline{\xi(t)\xi(t+\tau)}\, e^{i\omega\tau}\, d\tau \qquad (2)$$

$$= \beta g^2 \omega^{-5}$$

where ξ is the surface displacement and β is a dimensionless constant. Similarly, the two-dimensional wave-number spectrum

$$\Psi(\mathbf{k}) = (2\pi)^{-2} \iint_{-\infty}^{\infty} \overline{\xi(\mathbf{x})\xi(\mathbf{x}+\mathbf{r})} \cdot \exp(i\mathbf{k}\cdot\mathbf{r})\, d\mathbf{r} \qquad (3)$$

$$\propto k^{-4}$$

Over the last few years there has been good experimental support for the relations (2) and (3). Figure 2 shows some of the earliest results by *Burling* [1955] taken on Staines reservoir, Middlesex. The same relation, with the same value for β, has been found in the open ocean

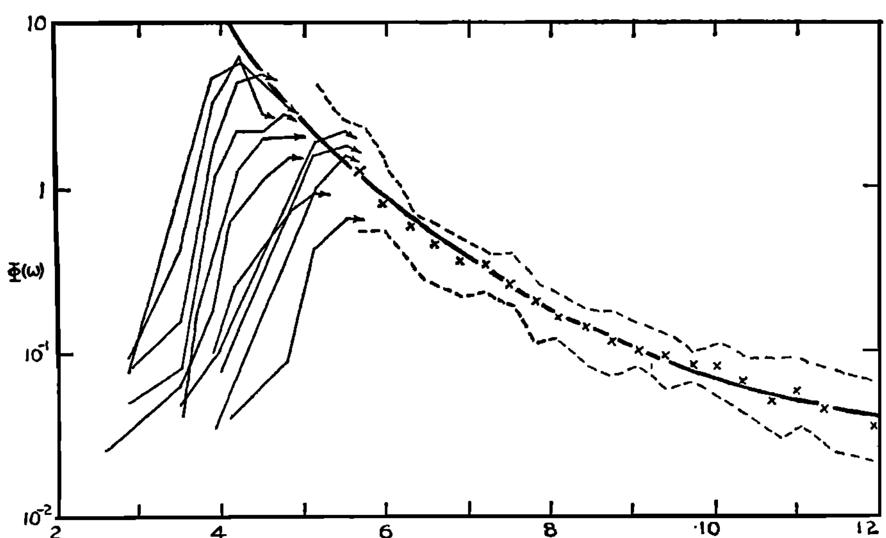

Fig. 2. Spectra of wind-generated waves measured by *Burling* [1955]. The cluster of lines on the left represents the spectra at low frequencies, for which saturation has not been attained. On the right the curves merge over the equilibrium range, and the broken lines indicate the extreme measured values of $\Phi(\omega)$ at each frequency ω. The crosses represent the mean observed value at each ω, and the heavy line the relation $\Phi(\omega) = \beta g^2 \omega^{-5}$ with $\beta = 7.4 \times 10^{-3}$.

as a result of the *Stereo-Wave Observation Project* [1957] and by the National Institute of Oceanography group [*Longuet-Higgins, Cartwright, and Smith,* 1961]. Other small-scale experiments are those of *Cox* [1958] and the extensive observations of *Hicks* [1961].

In summary, then, it appears that the relations (2) and (3) provide upper limit asymptotes to the wave growth; that the amplitude increases with time as described in section 2 until this upper limit is approximately attained, when further growth is inhibited. More accurately, the energy input from the waves is balanced by losses through wave breaking, and also, to some extent, by the nonlinear energy transfer mechanism described by *Hasselmann* [1962].

4. *Observational implications.* Let us consider briefly the implications of these theories to the observation of ocean waves, to what extent the predictions are borne out by present measurements, and what new observations might be made to provide further tests and further insight.

We will return for a moment to the generation process. It has frequently been noticed that a most striking property of the frequency spectrum $\Phi(\omega)$ is the steep forward face as the spectrum rises quite abruptly from a fairly low level almost to the saturation value. If we observe the wave spectrum at later and later times in the same wind field, the frequency of the steep forward face decreases. To put the same thing another way, if we look at a particular frequency in each of our records, then at a short wind duration, the energy content is quite low until suddenly it shoots up as the steep forward face moves past our observation frequency to lower values. Now this is just the behavior predicted by the theory, with its initial slow rate of growth under the influence of the resonance mechanism until the instability is triggered off and the rate of growth increases rapidly. It makes almost inevitable an identification between the frequency at which the steep forward face of the spectrum is found at a particular duration and the frequency undergoing transition in the sense that we have described.

As a first test of this conclusion, a number of observational spectra were examined by *Phillips and Katz* [1961], and the frequencies of the steep forward faces were compared with the transition frequencies, calculated theoretically. The results are shown in Figure 1, and it is striking that most of the experimental points lie on or to the left of the calculated curve, supporting our expectation that the transition occurs not later than the theoretical value. For very small values of c/U, the calculated curve is quite sensitive to small changes in the wind velocity profile, and the apparent agreement here between the theory and experiment is partly fortuitous, since the velocity profile in Pierson's observations was not measured.

This comparison is promising, but hardly convincing, since we are forced to attribute the premature transition to a more rapid rate of growth to the existence of an initial wave state of low energy, which, though undoubtedly present, was not measured. However, we can make a further prediction that should provide a more critical test of the interplay between the two mechanisms envisaged in the theory. This concerns the directional distribution of the wave components at frequencies near the frequencies of the spectral maximum, when $c < U$. According to the theory, the transition occurs for a given frequency first for the components traveling in the direction of the wind, so that the observed frequency of the steep forward face of the spectrum $\Phi(\omega)$ is strictly the transition frequency for components traveling in the wind direction. Components with the same frequency but traveling at an angle to the wind will not yet have undergone transition, so that the directional distribution of frequency components near the spectral peak should be strongly oriented toward the wind direction. At a slightly higher frequency, transition will have occurred over a range of angles, and the directional distribution should be much broader. Some of the results of *Longuet-Higgins, Cartwright, and Smith* [1961] suggest that this is so, as do casual observations at very short fetch and duration, but definitive measurements have yet to be made.

If the fetch and duration are large, an appreciable part of the wave energy may be contained by components traveling at about the same speed as the wind, i.e., for which $c \approx U$. As we have seen, the instability mechanism is ineffective under these conditions, so that these wave components have presumably acquired their energy by the resonance mechanism and should therefore show directional maxima at angles $\pm\cos^{-1}(c/U)$ to the wind. Figure 3 shows some results from the *Stereo-Wave Observation Project* [1957] that are relevant here. Each histogram gives the directional distribution of wave energy for a given frequency, and the black arrows show where maxima are expected theoretically. A disadvantage of these observations is that the meteorological conditions were not well defined and there are some uncertainties in the wind field in the generating area. However, the bimodal distribution is quite striking and is in accord with our theoretical expectations. Further evidence for the existence of directional maxima when $c \approx U$ is afforded again by *Longuet-Higgins, Cartwright, and Smith* [1961] in their experiments on a pitch-and-roll buoy.

Finally, the existence of the steep forward face at frequency ω_m, say, and the equilibrium range over frequencies $\omega > \omega_m$, suggest that we can reasonably approximate the spectrum by

$$\begin{aligned}\Phi(\omega) &= 0 \qquad 0 < \omega < \omega_m \\ &= \beta g^2 \omega^{-5} \qquad \omega > \omega_m\end{aligned}$$

so that the mean square surface displacement

$$\begin{aligned}\overline{\xi^2} &= 2 \int_0^\infty \Phi(\omega)\, d\omega \\ &= 2\beta g^2 \int_{\omega_m}^\infty \omega^{-5}\, d\omega \\ &= \tfrac{1}{2}\beta g^2 \omega_m^{-4}\end{aligned}$$

where ω_m, the frequency of the steep forward face, is very nearly equal to the frequency of the spectral peak, or the dominant frequency

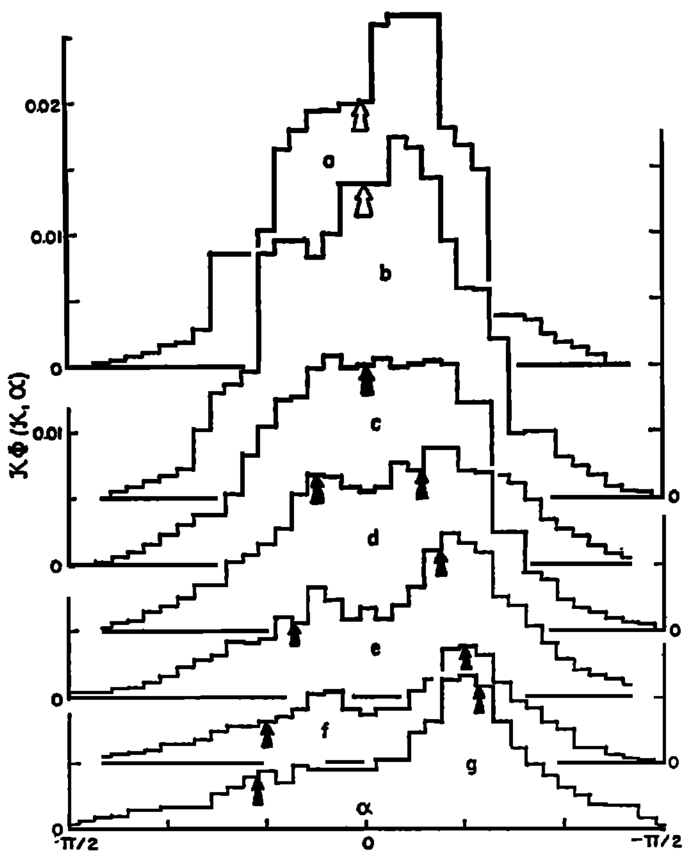

Fig. 3. Sections of the wave spectra $\Psi(\mathbf{k}) = \Psi(k, \alpha)$ (in polar coordinates) for fixed wave numbers k, observed in the Stereo-Wave Observation Project. The wavelengths, in meters, associated with each section are: (a) $\lambda = 118$, (b) $\lambda = 100$, (c) $\lambda = 73$, (d) $\lambda = 64$, (e) $\lambda = 56$, (f) $\lambda = 49$, (g) $\lambda = 44$.

of the wave field. A relation of this kind was discovered empirically by *Hicks* [1961] in his observations of wind waves on a pond.

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