The Budget of Turbulent Energy in the Lowest 100 Meters[1]

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Abstract. The budget of kinetic energy is computed for several periods in the lowest 100 meters. It is shown that the flux of energy is everywhere upward and that the divergence of energy flux is an important term in the energy budget at large negative Richardson numbers. Under these conditions the flux is produced mostly by low-frequency turbulence.

The dynamical equation governing the budget of the kinetic energy of atmospheric turbulence has been derived and discussed by *Richardson* [1920], *Rossby* [1926], *Calder* [1949], and *Blackadar* [1950], among others. The equation can be written in the form

$$\frac{d\bar{E}}{dt} = -\overline{u_i u_j}\,\frac{\partial U_i}{\partial x_j}(1 - Rf) - \frac{\partial \overline{Eu_i}}{\partial x_i} - \frac{1}{\rho}\frac{\partial(\overline{pu_j})}{\partial x_j} - \epsilon \qquad (1)$$

In this equation, u_i and u_j are the turbulent velocity components in Cartesian coordinates x_i and x_j, U_i is the mean velocity, Rf is the flux Richardson number, p the pressure, ϵ the rate of energy dissipation, and E stands for $u_i u_i/2$; a bar denotes a time average. The flux Richardson number should, in principle, contain a term depending on the vertical flux of moisture in addition to a term containing the vertical flux of sensible heat. Variations of density have been neglected in equation 1 except in the buoyancy term.

In the description of the state of flow in the lowest 100 meters it is customary to deal with coordinates x, y, and z so that x is directed along the mean wind, y at right angles to the wind and to the left, and z upward. The corresponding turbulent velocity components are u, v, and w, and the mean flow is U. The mean of w over periods normally considered (here an hour) is assumed to be zero. Also, horizontal gradients of the statistics of turbulence are usually assumed to be small compared with vertical gradients, so that we may write

$$\frac{d\bar{E}}{dt} = -\overline{u'w'}\,\frac{\partial U}{\partial z}(1 - Rf) - \frac{\partial \overline{Ew}}{\partial z} - \frac{1}{\rho}\frac{\partial \overline{pw}}{\partial z} - \epsilon \qquad (2)$$

Taylor [1952] has studied the magnitude of the terms in equation 2 at low levels (four cases at 2 meters and one at 29 meters) and has found that the only terms of importance were the rate of production of mechanical energy $-\overline{u'w'}(\partial U/\partial z)$ and the dissipation term ϵ. He therefore concluded that dissipation can be determined by measuring the product of Reynolds stress and wind shear. With increasing height, both decrease rapidly. On the other hand, the buoyancy term is nearly invariant with height, and, as will be seen, the same is true for the divergence of vertical energy flux. Therefore, it is the purpose of this paper to assess the relative magnitudes of the various terms at somewhat higher levels, from heights of about 25 to about 100 meters.

The observations for this study come from the 125-meter tower of the Brookhaven National Laboratory at Upton, N. Y. This tower, in a clearing surrounded by scrub pine, contains aerovanes and bidirectional vanes at 23, 46, and 91 meters, additional aerovanes at 11, 109, and 125 meters, and thermohms at all six levels. For a considerable number of hour-long periods, 5-second averages of the velocity components have been computed (3⅓ seconds in one case), and various statistics have been computed from

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them. These statistics include variances, covariances, spectra, and cross spectra, which have been published elsewhere (see, for example, *Panofsky and McCormick* [1954]).

Of the terms in equation 2, the quantity $-(1/\rho)/(\partial \overline{pw}/\partial z)$ has not been measured, since pressure fluctuations were not available. The term is usually regarded as small, since the spectrum of pressure [*Gossard*, 1960] contains relatively little variance in the frequency range of convective and mechanical turbulence.

The change of energy with time can be divided into a term expressing the local change and a term expressing advection by the mean flow. The magnitude of the local change can be computed for cases in which observations were made in successive hours. The magnitude of the change is 1 $m^2\ sec^{-2}\ hr^{-1}$, which is 3 ergs $g^{-1}\ sec^{-1}$, a quantity negligible compared with the other terms in equation 1. The advection term has not been measured but presumably is equally small. However, this assumption may well be questioned, since the ground at Brookhaven is not uniformly rough. In the same way it may not be considered legitimate to neglect the horizontal diffusion terms, as has been done in the derivation of equation 2 from equation 1.

Next, we shall consider the vertical flux of energy per unit density, $\overline{wE}$. This quantity will be called the vertical energy flux. Table 1 summarizes some measurements of it at three levels, along with the simultaneous Richardson numbers, based on average temperature and wind gradients between 11 and 125 meters (except that 91 and 23 meters were used for period 9, for which the winds at the other two levels were in doubt).

TABLE 1. Richardson Number and Vertical Flux of Kinetic Energy per Unit Density in $m^3\ sec^{-3}$

Period	Richardson Number	Energy Flux 23 m	46 m	91 m
G	−0.46			0.99
H	+0.03			0.34
L	+0.04			0.14
2	−0.03	0.35		0.19
3	−0.12	0.21	0.23	0.55
9	−0.60	0.27		0.81
11	−0.33	0.11		0.92
13	−0.52	0.36	0.62	1.08
15	−0.53	0.16		0.36

Three features of the fluxes stand out:

1. All fluxes are positive (directed upward).
2. The fluxes increase with height, so that there is flux divergence at all levels except in one case. Whether the vertical variation of flux departs significantly from the linear cannot be judged from the scanty measurements. In that single case the flux divergence is presumably not significantly different from zero.
3. The flux and flux divergence are small when the Richardson number is near zero. It is suggested by the observations that the fluxes of energy increase with increasing flux of sensible heat.

Table 1 also shows that the divergence of energy flux contributes quantities of the order of 100 ergs $g^{-1}\ sec^{-1}$ to the energy budget for the unstable periods, which is of the same order of magnitude as the largest terms in equation 2.

Figure 1 further analyzes the vertical energy flux according to frequency for one of the unstable periods, period 13. This figure gives, at the top, the spectra of vertical velocity at the three levels. These spectra, which were published previously [*Panofsky and Van der Hoven*, 1956], show, for example, the shift of the spectra toward decreasing frequency with increasing height. At the bottom of the figure are shown the three cospectra between vertical velocity and turbulent energy. It is clear that the cospectra drop off with frequency more rapidly than the spectra, suggesting that the small eddies produce little upward flux of energy, showing also that the spacing of observations was sufficiently close to measure the flux of energy. Since small eddies are characteristic of mechanical turbulence, it appears that most of the vertical energy flux is produced by the convective eddies. This result is consistent with the observation demonstrated in Table 1 that in purely mechanical turbulence the upward flux of energy is small. That it is not zero agrees with the wind-tunnel results reported by *Townsend* [1956]. The existence of a large upward flux of energy in a convective regime has also been postulated by *Ball* [1960], who used it to explain the diurnal variation of the height of the inversion above the atmospheric boundary layer.

Table 2 presents an attempt to evaluate all

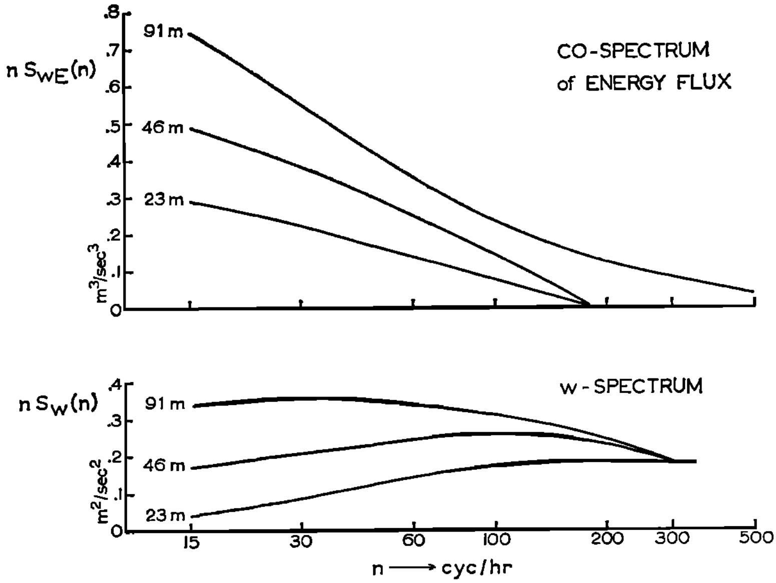

Fig. 1.

TABLE 2. Energy Budget, ergs g^{-1} sec^{-1}

Period	Average Production	Flux Divergence	Average Dissipation	Net Loss
G	316	108		
H	626	37		
L	507	15		
2	218	−24	162	138
3	188	50	101	151
9	189	79	46	125
11	284	119	72	191
13	194	106	72	178
15	67	29	32	61

the terms in equation 2 except the left-hand term and the pressure velocity correlation term. Columns 2 through 5 of this table show the energy budget averaged over the layer between 11 and 125 meters except 23 to 91 meters for period 3. The stress in the energy production term, assumed constant, was computed from the wind speed at 11 meters and the known roughness length, 1 meter, under the assumption of a logarithmic profile below 11 meters. Almost the same production term was obtained for cases for which Reynolds stresses were available at three levels by averaging these Reynolds stresses and multiplying by the mean shear. The flux Richardson number was replaced by the average gradient Richardson number between 11 and 125 meters. This replacement is likely to result in an underestimate in the rate of production of convective turbulence for the cases of large negative Richardson number. The sum of the rates of production for mechanical and convective turbulence is given in column 2 of Table 2. Column 3 shows the energy loss due to flux divergence. In the six cases in which energy fluxes were available both at 23 and at 91 meters the divergence term is based on these two fluxes. In the three in which only the flux at 91 meters was measured the divergence is shown under the assumption that the flux is a linear function of height and equal to zero at the ground. Comparing flux divergence and energy production rates, we see that divergence consumes a sizable part of the production for cases of large negative Richardson number—more than 50 per cent in one of them. In other words, a large fraction of the turbulence produced in the lowest 100 meters is exported upward and dissipated by doing work in the stable regions at higher levels. In the cases of small Richardson number, on the other hand, only a small fraction of the energy production goes into divergence. It is thus likely that the rate of creation of energy is a good measure of the energy dissipation when the Richardson number is small.

To complete the energy budget, an attempt was made to estimate independently the mean dissipation in the layer for the six cases in which spectra were available at 23, 46, and 91 meters. Spectra published by *Panofsky and Van der Hoven* [1956] and *Van der Hoven and Panofsky* [1954] were used for the u component of velocity. Since the spectral estimates are unprecise at the high-frequency end (at 360 cycles/hour) because of aliasing, those at 200 cycles/hour were used to estimate dissipation rates under the assumption of the $-5/3$ law of the inertial subrange. Now the frequency of 200 cycles/hour, which corresponds to a wavelength of about 100 meters or more, can hardly be claimed to belong in the inertial subrange. However, Van der Hoven and Panofsky as well as *Taylor* [1955] have shown that the $-5/3$ law for the longitudinal component extends to wave numbers below those of the inertial subrange. The equation for the spectral law is

$$S(k) = a\epsilon^{2/3}k^{-5/3} \tag{3}$$

where k is the wave number. The constant a for the one-dimensional longitudinal spectrum was derived from measurements by *Batchelor and Townsend* [1948] and found to be 0.33. The same result was used by *MacCready* [1953]. Other workers have found somewhat larger constants, e.g., 0.40 (R. J. Taylor) and 0.37 [*Kolmogoroff*, 1941]. R. W. Stewart reports later in this conference a constant of 0.43 as measured in the ocean. The use of this value would reduce the estimates of ϵ by 49 per cent. $S(k)$ was estimated from the observed time spectra under the assumption of Taylor's hypothesis that $k = n/U$, where n is the frequency. The observed spectra were corrected in two respects. First, the aerovane filters out high frequencies approximately according to the formula

$$S_1(n) = \frac{S(n)}{1 + (12\pi n/U)^2} \tag{4}$$

where S is the unfiltered spectrum, $S_1(n)$ the filtered spectrum; n is measured in cycles sec^{-1} and U in m sec^{-1}. The correction for this effect was of the order of 10 per cent.

Second, the original observations were averaged over 5 seconds. This gives rise to the relation

$$S_2(n) = S_1(n)\left(\frac{\sin 5\pi n}{5\pi n}\right)^2 \tag{5}$$

where $S_2(n)$ is the spectrum of the 5-second average and n, again, is in cycles sec^{-1}. Thus, the spectrum $S(n)$ is given by

$$S(n) = S_2(n)\left(\frac{5\pi n}{\sin 5\pi n}\right)^2\left[1 + \left(\frac{12\pi n}{U}\right)^2\right] \tag{6}$$

Table 3 shows the dissipation rates. As was to be expected, those derived from the spectra decreased rapidly with height; also, those at 91 meters vary approximately as the cube of wind speed.

Table 3 also gives the Kolmogoroff dissipation lengths; they are of the order of 1 mm.

TABLE 3. Dissipation Rates, ergs g^{-1} sec^{-1}, and Kolmogoroff Dissipation Lengths, mm

	Dissipation Rates			Dissipation Lengths		
Period	23 m	46 m	91 m	23 m	46 m	91 m
G			17			1.20
H			198			0.65
L			77			0.82
2	377	86	23	0.55	0.80	1.12
3	191	78	33	0.66	0.82	1.02
9	58	51	29	0.89	0.92	1.05
11	126	57	35	0.73	0.89	1.00
13	127	54	35	0.73	0.90	1.00
15	59	31	7	0.88	1.03	1.50

The arithmetic averages of dissipation for the three levels are given in column 4 of Table 2. Column 5 shows the net loss of energy, which is the sum of the two previous columns. The agreement with the production rates is fair, perhaps as good as could be expected in view of the uncertainty of the measurements and the omission of the terms produced by horizontal variation of turbulence statistics. However, the production exceeds the loss systematically, suggesting some systematic error in the estimates or a systematic behavior of the omitted terms.

Summary. The vertical flux divergence is an important term in the energy budget at large negative Richardson numbers but relatively unimportant for small Richardson numbers. In consequence, this term can also be neglected close to the surface. The flux of kinetic energy is everywhere upward, even with numerically small Richardson numbers.

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