# Aerodynamic Roughness of the Sea[1]

E. L. DEACON

*Division of Meteorological Physics*
*Commonwealth Scientific and Industrial Research Organization*
*Aspendale, Victoria, Australia*

*Abstract.* Observations of the vertical gradients of wind and temperature over the sea are presented. They indicate a drag coefficient for neutral stability increasing but slightly up to wind speeds of 14 m/sec. The mean value for the open sea is found to be approximately 0.0012 (for 10-meter reference height), but the number of observations on which this is based is much smaller than for the enclosed waters of Port Phillip. Analysis of the effects of stability on the wind gradient in terms of similarity theory gives some evidence that the concepts developed as a result of work over land surfaces may be applied successfully to conditions over the sea. The constant $\alpha$ in the 'log + linear' formulation for near-neutral profiles is found to have a value of around 3.5, a value similar to that found from some series of observations over land surfaces.

## INTRODUCTION

To obtain sufficiently accurate observations of wind profiles over the sea for the drag of the wind on the water to be deduced is no easy task. However, an earlier study [*Deacon, Sheppard, and Webb,* 1956] showed that anemometers mounted with as good exposure as possible on a small ship gave promising results. The observations here discussed were obtained on two subsequent series of trials with the same schooner, the *Derwent Hunter* of the C.S.I.R.O Division of Fisheries and Oceanography. The greater body of data thus obtained makes it possible to study the effect of vertical temperature gradients on the wind profiles, a study that could not be attempted in the earlier work. Not only is this of interest in paving the way toward a better knowledge of the variation of drag coefficients with stability, but also, by comparison with results over land, some light is thrown on the extent to which we are justified in applying relationships derived for solid surfaces to the flow over water disturbed by waves.

[1] Based on a paper presented at the International Symposium on Fundamental Problems in Turbulence and Their Relation to Geophysics sponsored by the International Union of Geodesy and Geophysics and the International Union of Theoretical and Applied Mechanics, held September 4–9, 1961, in Marseilles, France.

## OBSERVATIONAL METHOD

We concentrated effort on obtaining as accurately as possible the wind speed difference between two heights, 13 and 4 meters. These were chosen as high enough to avoid as far as possible uncertainties associated with conditions close above the wave tops. Accordingly two anemometers were mounted at 13 meters on the foremast crosstrees and two (often three) anemometers at heights between 3 and 6.4 meters on arms extending from a mast fitted near the end of the jib boom. The anemometers were calibrated in a wind tunnel before and after each trial period and also halfway through. To reduce errors still further, they were systematically interchanged after each day's work.

As was shown in the earlier work the effect of the ship's hull on the wind flow causes the lower anemometers to experience too low a wind speed by amounts of around 2 per cent. The magnitude of the correction to make for this was established, as before, by taking observations with the ship running at as low a speed as possible directly into wind and then at full speed (3.5 m/sec) into wind. If conditions are steady, the difference in the ratio of the wind at two heights as between the two runs enables a corrected ratio to be obtained. The hull effect corrections so found were nearly the same as (slightly smaller than) previously reported.

The speed of the ship relative to the water was measured by a propeller-type meter mounted beneath a float outrigged 3 meters from the side of the ship. The propeller actuated an electric counter alongside those actuated by the anemometers.

The rolling of the ship during runs was recorded, and the wind velocities were corrected for rolling in the manner indicated in the earlier paper. The corrections were appreciable only for the runs in the open sea; in Port Phillip, where a greater part of the observations were secured, swell is excluded by the narrow entrance and rolling was relatively slight.

Temperature differences between 13 and 4 meters were recorded and also the difference in temperature between sea surface and the air.

## Results

Observations obtained under rapidly varying conditions have been rejected as have a few observations where the fetch of the wind over deep water (15–20 meters) was less than 4 km. The remainder, 186 in number, have been grouped for wind speed at 10-meter height ($u_{10}$); the values of $(u_{13} - u_4)/u_{10}$ for some typical ranges are shown in Figure 1 plotted against the difference in temperature between air and sea. For the lowest two wind-speed ranges, curves have been drawn by eye to fit the points (as $A$ of Fig. 1), but for the rest straight lines have been fitted by the method of least squares to the Port Phillip data (examples $B$ and $C$ of Fig. 1). In each wind-speed range the open-sea values are rather consistently greater than those observed in Port Phillip, where the fetch of the wind over water was mainly from 12 to 30 km.

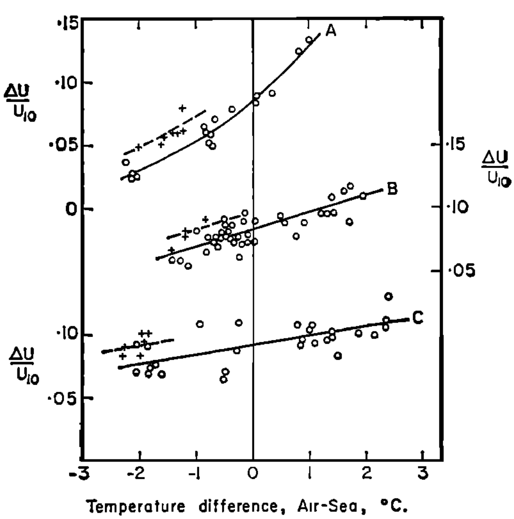


Fig. 1. The difference in wind speed ($\Delta u$) between 13 and 4 meters expressed as a fraction of 10-meter wind speed ($u_{10}$). $A$, for $u_{10}$ from 4.0 to 5.9 m/sec; $B$, for $u_{10}$ from 8.0 to 9.9 m/sec; $C$, for $u_{10}$ from 12.0 to 15.0 m/sec. Circles, Port Phillip; crosses, open sea.

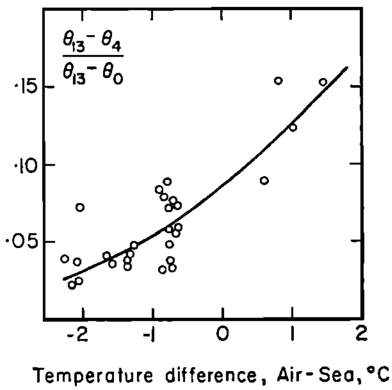


Fig. 2. The difference in potential temperature between 13 and 4 meters expressed as a fraction of the difference between 13 meters and the sea surface. Range of wind speed $u_{10}$ = 4.0 to 5.9 m/sec.

An analogous quantity in terms of potential temperature differences is $(\theta_{13} - \theta_4)/(\theta_{13} - \theta_0)$, where $\theta_0$ is the potential temperature of the sea surface. Figure 2 shows this plotted against the air-sea temperature difference for one of the wind-speed groups. The graphs are generally similar to those of Figure 1, but the scatter of points is greater.

*Drag coefficients for neutral stability.* From the values of $(u_{13} - u_4)/u_{10}$ for adiabatic conditions read from the curves of Figure 1 the values of the drag coefficient $c_{10} = \tau/(\rho u_{10}^2)$ have been calculated assuming the well-known logarithmic relationship

$$u_2 - u_1 = u_*/k \ln (z_2/z_1) \qquad (1)$$

in which $u_* = (\tau/\rho)^{1/2}$, and $k$, von Kármán's constant, has been taken to be 0.41. The resulting values are shown in Figure 3, which shows that up to a wind speed of about 9 m/sec the drag coefficients for Port Phillip are close to what laboratory researches lead us to expect for an aerodynamically smooth surface. At higher wind speeds there is a rise of drag coefficient, but only to the extent of 30 per cent or so at 14 m/sec.

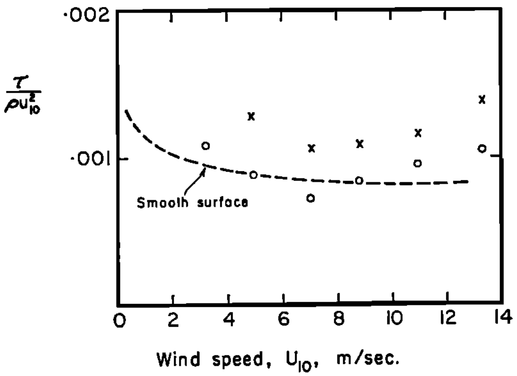


Fig. 3. Drag coefficient for neutral stability. Circles, Port Phillip; crosses, open sea.

The values for the open sea are estimated from the observations under unstable conditions, since equality of sea and air temperatures did not occur there during these trials.

The small magnitude of the variation in drag coefficient has also been found by Brocks from wind profiles obtained using a special buoy (*Brocks* [1959] and private communication). As a result of extensive trials in a bay of the Baltic (fetch about 20 km) he found little variation from a value $c_{10} = 0.0015$ over a similar range of wind speed, while North Sea trials near Heligoland gave a similarly near-constant value of $c_{10} = 0.0011$ for very much larger fetches of the wind over water—the full width of the North Sea.

*Fleagle, Deardorff, and Badgley* [1958] from observations over an inlet of the sea (fetch of wind over water about 10 km) found a value of $c_{10} = 0.0011$ and again little evidence of any variation over the range 3–9 m/sec. There is therefore general agreement among these three sets of data on the near constancy of the drag coefficient over an extensive range of wind speed.

For relatively enclosed waters (fetches 10–30 km) the $c_{10}$ values average 0.0012 (range $\pm 25$ per cent). Topographical differences between the various sites (both above and below water level) may be at least partly responsible for the different drag coefficients. The nature of the sea surface exhibits considerable variation in land-locked waters.

The rather tentative open-sea value of 0.0012 found in the present work is about 10 per cent greater than Brocks' result for the North Sea. The difference is hardly significant in view of the differences in technique and the small number of *Derwent Hunter* observations. It is, however, somewhat surprising that the difference between open sea and land-locked waters is of opposite sign in the two cases. It is possible that the rather great motion of the ship in the heavy southern ocean swell commonly experienced in the sea outside Port Phillip has resulted in a rather large experimental error owing to inadequacies in the corrections applied.

The new *Derwent Hunter* trial results are generally lower than those reported in the earlier paper, particularly at wind speeds above 8 m/sec. The open-sea near-neutral observations in the earlier trials gave $c_{10} = 0.0021$ for wind speeds from 10 to 14 m/sec. The new results should be more reliable, but the reason for the difference is not fully understood.

However, it seems evident now that there is no marked increase in drag coefficient at around 7 m/sec as at one time seemed plausible from early results.

*Effect of thermal stratification.* That the influence of thermal stratification on the wind profiles is considerable is shown in Figure 1: a more than 4-fold variation of $(u_{18} - u_4)/u_{10}$ with air-sea temperature difference is found in the lighter winds.

The influence of a turbulent heat flux $H$ through the layer of effectively constant shear stress requires a stability parameter to be included in generalized formulations of the wind and temperature profiles. A form of Richardson number useful for this purpose is that proposed by *Obukhov* [1946]; it relates height above the surface to a scale length $L$ expressed by

$$L = -u_* T_0 c_p \rho / kgH$$

in which $T_0$ is absolute temperature, $c_p$ and $\rho$ the specific heat at constant pressure and density of air, respectively, and $g$ the acceleration due to gravity. The generalized wind profile relationship for a suitably uniform level land surface is then

$$\partial u/\partial z = u_*/kz \qquad \varphi(z/L) \qquad (2)$$

in which the function $\varphi(z/L)$ has the value unity under neutral conditions ($H = 0$; $L = \infty$) corresponding to the logarithmic law.

An accurate knowledge of the form of $\varphi(z/L)$ has yet to be acquired, but *Monin and Obukhov* [1954] have pointed out that for sufficiently small departures from neutrality it must be a valid approximation to write

$$\varphi(z/L) = 1 + \alpha(z/L) \qquad (3)$$

where $\alpha$ should be a universal constant capable of empirical determination. The value of $\alpha = 0.6$ put forward by *Monin and Obukhov* [1954] based on observations over steppe is, however, considerably at variance with the findings of other workers [*Taylor,* 1960; *Panofsky, Blackadar, and McVehil,* 1960], who find values nearly 10 times as great. One of the difficulties of evaluating $\alpha$ is that we are, in effect, studying small departures from linearity when $u$ is plotted against log $z$; any uncertainty in the origin for $z$, as can occur with natural surfaces, can lead to considerable error by introducing spurious curvatures. As this uncertainty in zero level is likely to be rather marked when working over waves, a method of analysis is outlined here which obviates the difficulty.

Integration of equation 2 with respect to $z$ using approximation 3 gives, between two levels $z_1$ and $z_2$,

$$u_2 - u_1 = \frac{u_*}{k} \ln \frac{z_2}{z_1} + \frac{\alpha u_*(z_2 - z_1)}{kL} \qquad (4)$$

which is often known as the 'log + linear' law. From the definition of $L$ we see that in the linear term of equation 4

$$\frac{u_*}{kL} = -\frac{g}{T_0} \cdot \frac{H/\rho c_p}{\tau/\rho}$$

Using the flux equations

$$H/\rho c_p = -K_h \, \partial\theta/\partial z$$

and

$$\tau/\rho = K_m \, \partial u/\partial z$$

in which $K_h$ and $K_m$ are the eddy transfer coefficients for heat and momentum, respectively, we obtain

$$\frac{u_*}{kL} = \frac{g}{T_0}\left(\frac{K_h}{K_m}\right)\frac{\partial\theta/\partial z}{\partial u/\partial z}$$

If under near-neutral conditions $K_h/K_m$ is a constant $\gamma$, the vertical profiles of $\theta$ and $u$ will be similar, and the ratio of gradients can be replaced by the ratio of differences $\Delta\theta$, $\Delta u$ observed between two convenient heights.

Equation 4 may then be rewritten as

$$u_2 - u_1 = \frac{u_*}{k} \ln \frac{z_2}{z_1} + \frac{g}{T_0} \alpha\gamma \frac{\Delta\theta}{\Delta u}(z_2 - z_1) \qquad (5)$$

Without measurements of the fluxes, profile data only permit the evaluation of the product $\alpha\gamma$. For near-neutral conditions it is often assumed that $\gamma = 1$, and experimental determinations have not as yet established with certainty any departure from unity. In the following, $\alpha_p$ will be written for the product $\alpha\gamma$, so that no confusion should arise.

At small heights the stability term is negligible, and for a surface with rigid roughness elements it follows that, if $u_e$ is the wind velocity at a small height, $u_*/u_e$ is a constant. (At very low wind speeds the Reynolds number of the flow about the roughness elements may be too low for the square law of resistance to apply, but in the material dealt with here this is not so.) For a given pair of heights the first term on the right-hand side of equation 5 can be rendered constant by dividing the equation throughout by $u_e$, giving

$$\frac{u_2 - u_1}{u_e} = \frac{u_*}{ku_e} \ln \frac{z_2}{z_1} + \frac{g\alpha_p \, \Delta\theta(z_2 - z_1)}{T_0 u_e \, \Delta u} \qquad (6)$$

If, therefore, we have a series of observations of the wind speeds at heights $z_1$, $z_2$, and $z_e$ made with a variety of $\Delta\theta/\Delta u$ values, then by plotting the left-hand side of equation 6 against

$$S \equiv \frac{g \, \Delta\theta(z_2 - z_1)}{T_0 \, \Delta u \cdot u_e}$$

we should obtain points falling close to a curve which, over the near-neutral range, should approximate to a straight line of slope $\alpha_p$. As will be seen from equation 6, the value of $\alpha_p$ obtained by this method is independent of the datum level from which heights are measured, as long as this is constant for a series of observations.

*Example of analysis: land data. Taylor* [1960] has shown that the range of stability over which the log + linear law is a valid approximation is quite small. We therefore need a series of observations that fall mainly in or

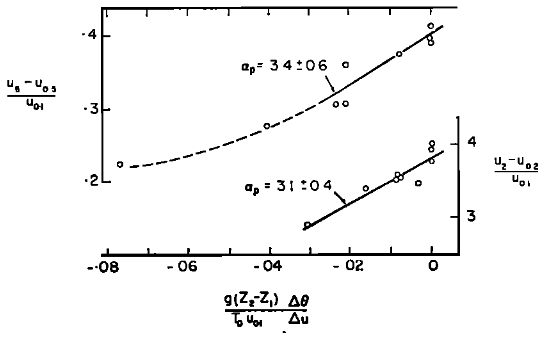


Fig. 4. Wind data of Calder for a desert site plotted to give $\alpha_p$ in the log + linear profile relationship.

close to this range. A good series for this purpose was secured by *Calder* [1943] over a very level desert surface where steady strong winds caused only small values of $S$ to be experienced; the author is indebted to Calder for the use of these unpublished data. The values of $\Delta\theta$ and $\Delta u$ are differences over the height interval 0.30 to 4.25 meters; for $u_e$ we may take the wind speeds measured at 0.1-meter height. In Figure 4 the results are shown for two different height intervals: for the one, $z_1$ and $z_2$ are 5 meters and 0.5 meters; for the other, 2 meters and 0.2 meters. The straight lines fitted by least squares over the ranges shown are

$$\text{For 5 and 0.5 m, } \alpha_p = 3.4 \pm 0.6$$

$$\text{For 2 and 0.2 m, } \alpha_p = 3.0 \pm 0.4$$

Observations by *Deacon* [1953] over grassland also give a value of $\alpha_p \sim 3$, so that further support is given to *Taylor's* [1960] conclusion that the value of 0.6 given by *Monin and Obukhov* [1954] is much too small.

*Application to observations over the sea.* Over the sea the justification for assuming $u_*/u_e$ to be constant is not so evident as for a surface with rigid roughness elements. It is however of interest to apply the above method of analysis to the observations of *Fleagle, Deardorff, and Badgley* [1958], which lend themselves to this purpose since wind speeds were observed at a height as low as 0.31 meter. These are taken for $u_e$ and $\Delta\theta$; $\Delta u$ are for the height interval 0.4 to 3.2 meters. Figure 5 shows the plotting for $z_1 = 1.33$ meters, $z_2 = 4.42$ meters. The tangent drawn to the curve at the neutral stability point gives $\alpha_p = 3.6$. The winds at other pairs of levels were treated similarly, and the results are:

| $z_1$ | | $z_2$ | $\alpha_p$ |
|---|---|---|---|
| 1.33 | and | 4.42 m | 3.6 |
| 0.91 | and | 2.91 m | 3.7 |
| 0.60 | and | 2.13 m | 3.8 |
| | | Mean | 3 7 |

Examination of the points in these graphs shows no tendency for them to separate themselves in any systematic fashion depending on wind speed.

The intercept at neutral stability in Figure 5 is some 50 per cent smaller than is consistent with the complete neutral-wind profile. This is caused by systematic errors in the wind speeds, as discussed by Fleagle, Deardorff, and Badgley. However, using wind speeds at well separated levels, the value of $\alpha_p$ so derived is rather insensitive to such systematic errors.

In the *Derwent Hunter* trials wind speeds close to the water surface were not observed, and the evaluation of $\alpha_p$ from the 13- to 4-meter wind and temperature differences is rather more complicated. However, by taking trial values of $\alpha_p$ and using them in the log + linear law to calculate $u_*/u_e$, it is possible to find the value corresponding to constant $u_*/u_e$ in the near-neutral region. The values so found, to the nearest half unit, are:

| Range of $u_{10}$, m/sec | 4–6 | 6–8 | 8–10 | 10–12 | 12–15 |
|---|---|---|---|---|---|
| Value of $\alpha_p$ | 3.0 | 3.0 | 3.0 | 3.5 | 4.0 |

For the four highest wind-speed ranges the straight lines fitted by least squares to the ob-

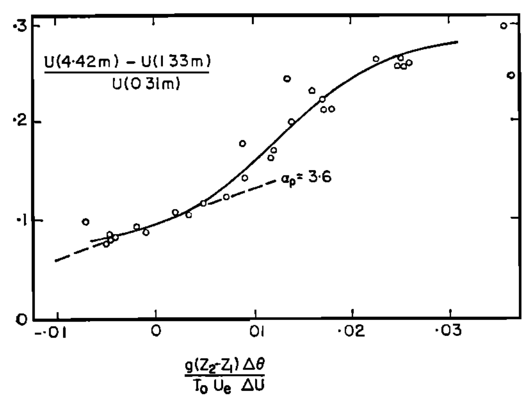


Fig. 5. Wind profile data of *Fleagle, Deardorff, and Badgley* [1958] plotted to give $\alpha_p$ in the log + linear relationship.

servations (as illustrated in Fig. 1, *B* and *C*) were employed; for the lowest wind-speed range, the tangent to the curve at neutral stability. The mean $\alpha_p$ of 3.3 is in reasonably good agreement with that of 3.7 obtained from the data of Fleagle, Deardorff, and Badgley.

The similarity between the values of $\alpha_p$ over land and sea is reassuring. It gives some reason to suppose that the concepts developed as a result of work over the land may apply quite well to conditions over the sea.

## References


Brocks, K., Measurement of wind profiles over the sea and the drag at the sea surface, International Oceanographical Congress, New York, 1959.

Calder, K. L., Some measurements of the vertical gradient of wind speed and air temperature over a flat desert surface, *Chem. Defence Research Estab., India, Rept. 268,* 1943.

Deacon, E. L., Vertical profiles of mean wind in the surface layers of the atmosphere, *Geophys. Mem. 91,* Air Ministry Meteorol. Office, London, 1953.

Deacon, E. L., P. A. Sheppard, and E. K. Webb, Wind profiles over the sea and the drag at the sea surface, *Australian J. Phys., 9,* 511–541, 1956.

Fleagle, R. G., J. W. Deardorff, and F. I. Badgley, Vertical distribution of wind speed, temperature and humidity above a water surface, *J. Marine Research, 17,* 141–157, 1958.

Monin, A. S., and A. M. Obukhov, Basic regularity in turbulent mixing in the surface layers of the atmosphere, *Akad. Sci., SSSR, Trudy Geofiz. Inst., 24,* 1954.

Obukhov, A. M., Turbulence in an atmosphere of nonhomogeneous temperature, *Trudy Inst. Theoret. Geofiz. Acad. Nauk SSSR, 1,* 95–115, 1946.

Panofsky, H. A., A. K. Blackadar, and G. E. McVehil, The diabatic wind profile, *Quart. J. Roy. Meteorol. Soc., 86,* 390–398, 1960.

Taylor, R. J., Similarity theory in the relation between fluxes and gradients in the lower atmosphere, *Quart. J. Roy. Meteorol. Soc., 86,* 67–78, 1960.