Determination of the Rate of Dissipation of Turbulent Energy near the Sea Surface in the Presence of Waves[1]

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Abstract. The hot-film anemometer and its associated electronics have now been brought to a high level of performance and reliability. Enough measurements of one-dimensional turbulent spectra have now been obtained to establish with sufficient reliability the form of the high-wave-number spectrum. According to the Kolmogoroff hypothesis, this form is universal in shape and its scaling parameters depend only upon the kinematic viscosity ν and the rate of energy decay per unit mass ϵ. Thus, when ν is known, measurement of any part of the spectrum permits determination of ϵ. In the presence of masking noise, an upper limit may be placed on ϵ. These concepts are illustrated with some measurements taken near the sea surface in the presence of waves.

Recent measurements of the spectrum of turbulence at very high Reynolds number in a tidal stream (*Grant, Stewart, and Moilliet* [1962], hereafter referred to as paper 1) have given strong support to the Kolmogoroff hypothesis. This hypothesis states that at sufficiently high wave number the statistical structure of turbulence has a universal form, and that the scaling parameters depend only upon ϵ, the rate of energy decay per unit mass, and upon ν, the kinematic viscosity.

The hypothesis implies that at high wave numbers the turbulence is isotropic. A one-dimensional spectrum is thus sufficient to describe the complete high-wave-number spectrum. We define the one-dimensional spectral energy density function $\varphi\ (k)$ by

$$\varphi(k) = L^{-1}\left\{\int_0^L u(x)e^{ikx}\,dx\right\}\cdot\left\{\int_0^L u(x)e^{-ikx}\,dx\right\} \qquad (1)$$

$$\int_0^\infty \varphi(k)\,dk = \overline{u^2} \qquad (2)$$

(This definition of $\varphi\ (k)$ is that used in paper 1 and by *Hinze* [1959], but is twice that used by *Batchelor* [1953] and by some other authors.) In these expressions, u is the turbulent velocity component in the direction of x, k is the wave number, and L is a length scale taken so that $L \gg k^{-1}$. According to the hypothesis, $\varphi\ (k)$ must be of the form

$$\varphi(k) = \epsilon^{1/4}\nu^{5/4}F(k\nu^{3/4}\epsilon^{-1/4}) = \epsilon^{1/4}\nu^{5/4}F(k/k_s) \qquad (3)$$

$F(k/k_s)$ is a universal function of its argument $k/k_s = k\ \nu^{3/4}\ \epsilon^{-1/4}$ and is common to all fields of turbulence for which the hypothesis is valid. There are no clear-cut rules delineating the range of validity of the hypothesis. However, it is likely to be applicable for wave numbers sufficiently large that their characteristic time constant is much less than the time required for the large-scale motions to change their statistical properties significantly. It is also necessary that the energy in these wave numbers be derived from larger-scale turbulence, and not directly from the mean flow field.

The measurements reported in paper 1 were obtained by towing a hot film flowmeter some 15 meters deep in a region of very intense turbulence but very small surface waves. The properties of this turbulence should closely meet the condition for validity of the Kolmogoroff hy-

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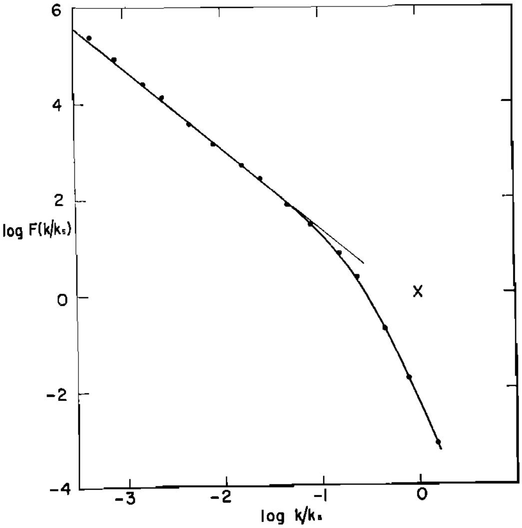

Fig. 1. The normalized spectrum function $F(k/k_s)$.

pothesis, and the measured spectra agree fairly well, but not exactly, with those calculated by *Reid* [1960] from the theories of *Heisenberg* [1948] and of *Kovasznay* [1948]. The reasoning leading to the theoretical spectra is not sufficiently compelling to cause us to prefer either to the experimental ones, so we have chosen as our best estimate of $F(k/k_s)$ what we consider to be the best of the experimental spectra reported in paper 1. This spectrum is reproduced in Figure 1.

When the same apparatus is used to attempt turbulence measurements in the presence of waves, or at much lower intensities, difficulties arise. The signal-to-noise ratio decreases greatly, both because of a lower signal and because motion and vibration induced by the waves increase the noise. Also, of course, the apparatus responds to the orbital motion of the waves.

However, if we accept the results shown in Figure 1 as definitive of $F(k/k_s)$, and can assume the Kolmogoroff hypothesis for the portion of the spectrum being measured, then *any* measure of the spectral level at any wave number establishes an upper limit to ϵ. This is because no effects of the kind discussed above can reduce the measured value of φ (k). For a given ν, (3) describes a single family of curves. Any single value of φ (k) is sufficient to select one of these curves and therefore to define a particular value of ϵ. It is the chief purpose of this paper to demonstrate the use of a technique for determining an upper limit for ϵ, using measurements of turbulence in the presence of waves as the example.

It is evident that, if log φ is plotted against log k, all curves of the form (3) can be derived from one such curve by simple translations. Moreover, for given ν, a change in ϵ by a factor α results in a displacement of the curve by ¼ log α, both horizontally and vertically. In practice the following procedure is used:

A plot of log $F(k/k_s)$ versus log k/k_s is prepared, and the point (0, 0) is located and marked (Fig. 1). For the measured spectrum, a plot of log φ (k) versus log k is prepared on the same scale. Now if the measured spectrum is free of noise and nonturbulent signals, and if Kolmogoroff's hypothesis is valid for the measured turbulent field, the two plots can be made to coincide. When in coincidence, the point (0, 0) on the $F(k/k_s)$ curve will correspond to the point (log $\epsilon^{1/4}$ $\nu^{-3/4}$, log $\epsilon^{1/4}$ $\nu^{5/4}$) on the $\varphi(k)$ curve. For a given value of ν but varying values of ϵ, the locus of the point (0, 0) on the $F(k/k_s)$ curve will be a straight line of slope +1 through (log $\nu^{-3/4}$, log $\nu^{5/4}$).

If the measured $\varphi(k)$ spectrum contains noise or other spurious signals, the fit may nevertheless be attempted. A line of slope +1 is drawn through the point (log $\nu^{-3/4}$, log $\nu^{5/4}$), and the $F(k/k_s)$ curve is superimposed so that the point (0, 0) always lies on this line. With this constraint the position is sought for which no point on the $\varphi(k)$ curve falls within (to the lower left of) the $F(k/k_s)$ curve. The extreme position of the $F(k/k_s)$ curve under these conditions then defines the largest value of ϵ, ϵ_{max}, consistent

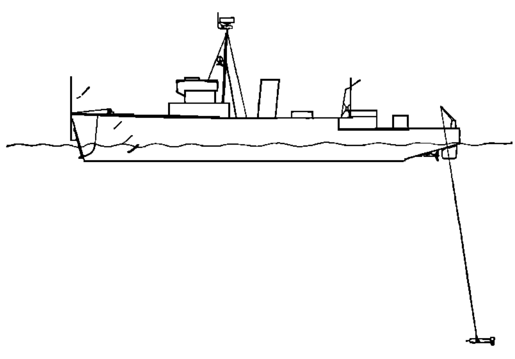

Fig. 2. The research vessel *C.N.A.V. Oshawa*, showing the probe mountings on the bow and on the towed body. The over-all length of the ship is 217 feet.

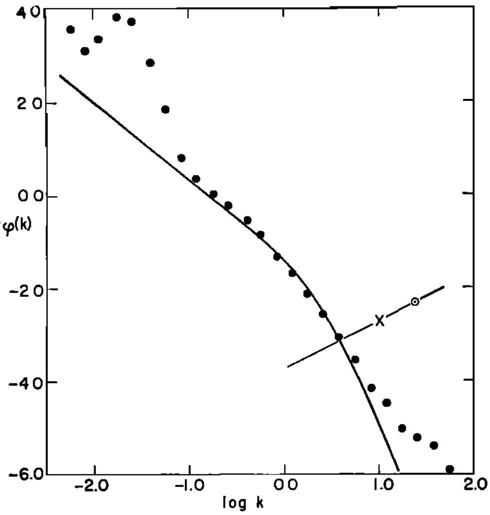

Fig. 3. The function $F(k/k_s)$ fitted to a measured spectrum in the prescribed manner.

with the measurements, provided that Kolmogoroff's hypothesis is valid for the measured field. The point coincident with (0, 0) is (log $\epsilon_{max}^{1/4}$ $\nu^{-3/4}$, log $\epsilon_{max}^{1/4}$ $\nu^{5/4}$), and hence ϵ_{max} may be calculated.

Measurements have been made in the presence of waves by means of the equipment described in paper 1. The measuring probe is a constant-temperature hot-film flowmeter, which in the absence of vibrations and large temperature fluctuations has a broad-band noise level corresponding to 10^{-2} cm/sec. The signal and its first derivative are recorded on a multichannel FM tape recorder, and spectral analysis is performed by an analogue technique upon return to the laboratory.

As was described in paper 1, for depths of 2 meters and less the probe was mounted over the bow of the research vessel *C.N.A.V. Oshawa*. For greater depths it is mounted in a body towed from the quarterdeck. Figure 2 is a sketch of the arrangements.

Measurements have been obtained on a few occasions in Georgia Strait and in Juan de Fuca Strait, near Vancouver Island, under some variety of wave conditions. These measurements are of a preliminary nature, and only rudimentary supporting data of wavelengths and heights and wind speeds were obtained by 'seaman's eye.'

Figure 3 shows a measured spectrum taken at a depth of 1.5 meters in the presence of waves estimated to be 5 meters long and 0.4 meters high (peak to trough) with a wind speed of about 6 m/sec. The large deviation from the form $F(k/k_s)$ at the left is consistent with the assumption that it is due to orbital velocities in the waves if allowance is made for the wave propagation.

(Although we have plotted the spectra as spatial, they were taken as time spectra, and Taylor's hypothesis was assumed [*Hinze,* 1959]. This assumption, justified for the turbulence, is not valid for the waves. The ship was normally steaming into the wind at about 1.5 m/sec. Waves of length 6 meters propagating at 3 m/sec then have an apparent period of 1.3 sec, and will be interpreted as motion of scale 2 meters with $k = 0.03$ cm^{-1}. This accounts for the location of the 'wave' peak in the spectrum at this apparently anomalous high wave number.)

The deviation at the right is due to noise. ϵ_{max} here is 0.023 erg cm^{-3} sec^{-1}.

Figure 4 shows the result when the same procedure is applied to a spectrum measured at a depth of 15 meters in Georgia Strait. ϵ_{max} is estimated to be 0.0011 erg cm^{-3} sec^{-1}. We have permitted one point to lie below our $F(k/k_s)$ line, as the possibility of errors (and mistakes) made it appear unwise to base the result on a single point. As can be seen in Figures 3 and 4, the region of 'fit' is in the neighborhood of $k = 1$ cm^{-1}, corresponding to a scale of the order of 6 cm. This is small scale relative to the depth

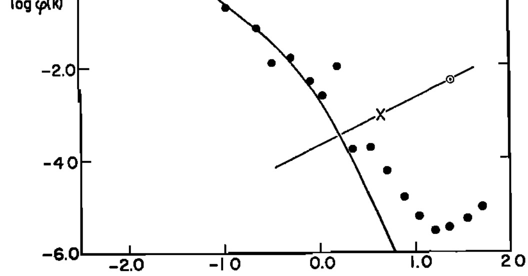

Fig. 4. The function $F(k/k_s)$ fitted to a 'noisy' measured spectrum that represents a very low rate of dissipation of turbulent energy.

of 1 or 2 meters, and there is a very good chance that the requirements for validity of Kolmogoroff's hypothesis are met.

TABLE 1. Energy Dissipation (ergs cm^{-3} sec^{-1}) as a Function of Depth below Surface and of Wave Height

Depth, meters	Wave Height, meters					
	0.10	0.20	0.30	0.40	0.50	0.90
1					0.042	
1.5		0.015		0.023		
2	0.0052		0.029		0.022	0.045
12					0.00025	
15	0.0011					

Table 1 shows a compilation of the data we now have on ϵ as a function of wave height and probe depth. The data are scanty as yet, but the rather weak dependence on depth near the surface and the expected increase of ϵ with wave height are noticeable. Our meteorological and wave data in these preliminary experiments leave much to be desired, but generally the waves were fetch-limited, with the shorter waves in equilibrium with winds of 5–10 m/sec.

It is interesting to compare the observed turbulent dissipation under the waves with the rate of energy input to waves. No really reliable method of determining the latter is available, but if we adopt the view of *Stewart* [1961] that a large proportion of the drag of air on water is wave drag, we may make some estimates. Taking wind speed U as 7 m/sec and guessing the wave drag coefficient to be 5×10^{-4}, we find the wave drag to be

$$\tau_w = c_d \rho_{air} U^2 = 0.3 \text{ dyne cm}^{-2} \qquad (4)$$

The rate of energy input will be given by this rate of momentum input multiplied by some mean wave phase speed. If we take this to be 3 m/sec, we find the rate of energy input to be $\sim$100 ergs cm^{-2} sec^{-1}. This is of course very rough, but it is difficult to see how it could be in error by a factor of as much as 4.

Of the wave energy, more than half is concentrated above the trough line (all the potential energy and some of the kinetic energy), and more than 90 per cent of the energy in a wave 6 meters long is located within 1 meter of the mean surface. It would seem reasonable to expect the dissipation in such a wave to be concentrated in the top meter, and we note from Table 1 that between 1 and 2 meters there is rather little variation in ϵ, the average being about 0.03 erg cm^{-3} sec^{-1}. Over 1 meter this yields about 3 ergs cm^{-2} sec^{-1}, which is a full order of magnitude less than our best estimate of input.

We are therefore led to infer that almost all wave dissipation is concentrated very near to the surface, essentially above the trough line. Presumably the mechanism is wave breaking, with the resulting 'splash turbulence' penetrating very little into the body of the fluid.

The small variation in ϵ with depth bears no resemblance to the variation in wave kinetic energy density (particularly in view of the fact that dimensional analysis suggests that ϵ should depend upon the 3/2 power of the energy density). It is thus tempting to suggest that the observed turbulence is more closely associated with the wind-driven drift current than with the waves.

The observations at depths of 12 and 15 meters are even more unlikely to have any connection with the waves. There were substantial density gradients at shallower depths, and the tidal currents are not negligible in these waters. The observed dissipation rates should therefore not be taken as typical of these depths in the open sea.

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References

Batchelor, G. K., *The Theory of Homogeneous Turbulence,* Cambridge University Press, London, 1953.

Grant, H. L., R. W. Stewart, and A. Moilliet, *J. Fluid Mech., 12,* 241–268, 1962.

Heisenberg, W., *Proc. Roy. Soc. London, A, 195,* 402, 1948.

Hinze, J. O., *Turbulence,* McGraw-Hill Book Co., New York, 1959.

Kovasznay, L. S. G., *J. Aero. Soc., 15,* 745, 1948.

Reid, W. H., *Phys. Fluids, 3,* 72–77, 1960.

Stewart, *J. Fluid Mech., 10,* 189–194, 1961.