On the Interpretation of Smoke Diffusion and Wind Analysis Data at Risø[1]

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Abstract. Wind velocity measurements and smoke patterns are compared. The most frequent situation is a Gaussian wind velocity fluctuation distribution, which is in good agreement with the theoretical picture developed by C. F. Wandel and the author. Occasionally, under conditions usually classified as typical inversion situations, definite non-Gaussian distributions are observed. In such cases diffusion is slow, and a low-frequency cutoff in the Lagrangian power spectrum is proposed for the description of the phenomenon.

Introduction. Smoke from the Kyndby power station and wind velocity data from the Risø micrometeorology tower [*Møller and Jensen*, [1959] have been compared. In the most frequent case of Gaussian wind velocity fluctuation distributions a good fit is obtained to the theoretical estimate of the Eulerian-Lagrangian transform derived by Wandel and the author. Occasionally, non-Gaussian distributions, combined with a very slow rate of diffusion, occur in connection with well-developed inversion conditions. The Gaussian case never leads to diffusion slower than $\overline{z^2} \propto t$. Under certain conditions, however, observations of $\overline{z^2} \propto t^{1/6}$ or $\overline{z^2} \simeq$ constant are reported [*Leonard*, 1958; *Hilst and Simpson*, 1958]. Such observations as well as our own investigations suggest the existence of a suppression of the low-frequency part of the Lagrangian power spectrum in the non-Gaussian case.

Observations. Smoke from the Kyndby power station is observed visually or photographically. When the smoke is sufficiently 'thin' relative to the background sky good estimates of $\overline{z^2}(t)$ can be obtained both visually and photographically. Several observations are performed for each distance in order to get a true mean, and time is calculated from the distance from the chimney and the mean wind speed $\bar{U}(z)$ at the mean smoke height. So far only vertical diffusion has been investigated, and the following magnitudes have been obtained from the micrometeorology tower. First, the mean wind speed $\bar{U}$ is observed as a function of height and a wind profile is obtained. Thus the value of $\bar{U}$ at the height of the smoke plume can be determined. Second, the vertical wind fluctuations $w(t)$ are observed and a distribution function $P(w)$ is obtained. The integral

$$\int_{-\infty}^{w} P(w)\, dw$$

is plotted on graph paper, representing a Gaussian by a straight line. From this graph the square root w' of the mean square deviation is obtained, and the distribution is inspected for consistency with a Gaussian. Third, w is recorded for a period of time very nearly the same as the travel time of the smoke. The record is made at a fast paper speed so that details of $w(t)$ are evident. The total recorded time interval is now subdivided into intervals of duration T'. From each of these intervals the average value $\langle w \rangle_{T'}$ is obtained, in which the subscript T indicates that the averages are taken over the limited period T'. In this way several values of $\langle w \rangle_{T'}$ are obtained, and we then find

$$\sigma_{1T'}^{2} = \overline{\langle w \rangle_{T'}^{2}} - \overline{\langle w \rangle_{T'}}^{2} \qquad (1)$$

where the subscripts 1 and T' indicate a measure of the mean square deviation of the *first* moment $\langle w \rangle_{T'}$ when measured over intervals of time T' and the bar indicates the mean value over the distribution of $\langle w \rangle_{T'}$.

Where $P(w)$ is consistent with a Gaussian the

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four measured magnitudes $\overline{z^2}(t)$, $\bar{U}$, w', and $\sigma_{1T'}{}^2$ constitute our basic data for a comparison of Lagrangian and Eulerian data.

Interpretation of Gaussian cases. Taylor's description of turbulent diffusion

$$\overline{z^2}(t) = 2w'^2 \int_0^t \int_0^{t'} R_L(\tau)\, d\tau\, dt' \qquad (2)$$

where $R_L(\tau)$ is the Lagrangian autocorrelation function, leads, when Fourier-transformed and integrated twice, to

$$\overline{z^2}(t) = 2w'^2 t \int_0^\infty \frac{\sin^2(\omega t/2)}{(\omega t/2)^2} P_L(\omega)\, d\left(\frac{\omega t}{2}\right) \qquad (3)$$

where $P_L(\omega)$ is the Lagrangian power spectrum normalized to unity. C. F. Wandel and the author have shown that under the conditions of homogeneous, isotropic, fully developed, and stationary turbulence the Eulerian and Lagrangian power spectra are connected with the scalar energy spectrum

$$E(k) = \tfrac{1}{3}E_\parallel(k) + \tfrac{2}{3}E_\perp(k) \qquad (4)$$

by the transformations

$$P_{E,\bar{U}}(\omega) = \int_0^\infty \frac{E(k)}{\bar{U}k} \frac{1}{\sqrt{\pi}} \frac{\bar{U}}{2u'} \cdot \left\{ \exp\left[-\frac{((\omega/\bar{U}k) - 1)^2}{2u'^2/\bar{U}^2}\right] - \exp\left[-\frac{((\omega/\bar{U}k) + 1)^2}{2u'^2/\bar{U}^2}\right]\right\} \frac{\omega}{\bar{U}k}\, dk \qquad (5)$$

and

$$P_L(\omega) = \int_0^\infty \frac{E(k)}{\sqrt{2}\, u'k} \sqrt{\frac{2}{\pi}} \left(\frac{\omega}{\sqrt{2}\, u'k}\right)^2 \cdot \exp\left[-\frac{\omega^2}{4u'^2k^2}\right] dk \qquad (6)$$

where $u' = v' = w'$.

It follows from these expressions that small values of k contribute to spectrum values for small ω values, and since, furthermore,

$$E(k) = E(0) + 0(k^2) \qquad (7)$$

we have the relations

$$P_{E,\bar{U}}(0) = \frac{E(0)}{\bar{U}} \phi\left(\frac{\bar{U}}{\sqrt{2}\, u'}\right) \qquad (8)$$

where $\phi(x)$ is the error function [*Jahnke and Emde*, 1945], and

$$P_L(0) = \frac{E(0)}{u'\sqrt{\pi}} \qquad (9)$$

The same arguments as used in deriving the transformations 5 and 6 can be used for a determination of $\sigma_{1T'}{}^2$, and it follows that

$$P_{E,\bar{U},\perp}(0) = \frac{1}{\pi w'^2} \sigma_{1T'}{}^2 \cdot T' \qquad (10)$$

Furthermore, for long-time diffusion we are left with Fickian diffusion, and it follows from equation 3 that

$$\overline{z^2}(t) \to \pi w'^2 t P_L(0) \qquad (11)$$

Although the observed turbulence is not strictly isotropic, we shall assume that order-of-magnitude checks on our relations can be obtained by assuming isotropy and using the relation 4; we obtain

$$P_{E,\bar{U}}(0) = \tfrac{4}{3} P_{E,\bar{U},\perp}(0) \qquad (12)$$

Consequently, by combining equations 8, 9, 10, 11, and 12 and substituting w' for u', we obtain

$$\overline{z^2}(t) \to \frac{4}{3\sqrt{\pi}} \sigma_{1T'}{}^2 T' \cdot t \frac{\bar{U}}{w'} (\phi(\bar{U}/\sqrt{2}\, w'))^{-1} \qquad (13)$$

For short-time diffusion we have

$$\overline{z^2}(t) \to w'^2 t^2 \qquad (14)$$

These two expressions can be brought to a common dimensionless form as follows. The characteristic length occurring in (13) and pertaining to k space is $E(0)$, which in terms of the measured Eulerian quantities is given by

$$\sqrt{\pi}\, E(0) = \Lambda \equiv \frac{4}{3\sqrt{\pi}} \bar{U}\sigma_{1T'}{}^2 \cdot T' \Big/ \left(w'^2 \phi\left(\frac{\bar{U}}{\sqrt{2}\, w'}\right)\right) \qquad (15)$$

Similarly the characteristic time occurring in (13) and pertaining to the Lagrangian power spectrum is $P_L(0)$, which in terms of the measured Eulerian quantities is given by

$$\pi P_L(0) = T \equiv \frac{4}{3\sqrt{\pi}} \bar{U}\sigma_{1T'}{}^2 \cdot T' \Big/ \left(w'^3 \phi\left(\frac{\bar{U}}{\sqrt{2}\, w'}\right)\right) \qquad (16)$$

If these characteristic magnitudes are used as

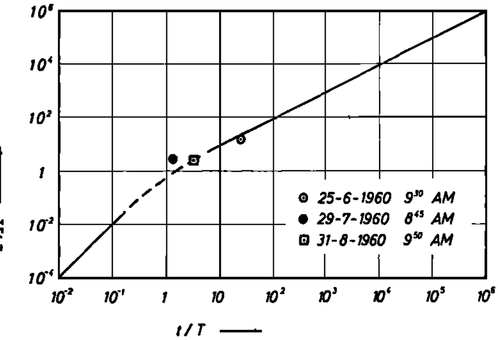

Fig. 1. Example of the universal diffusion pattern in homogeneous, isotropic, fully developed, and stationary turbulence.

units we find the following asymptotic diffusion expressions:

$$\frac{\overline{z^2}(t)}{\Lambda^2} \longrightarrow \frac{t}{T} \quad \text{for large} \quad t \tag{17}$$

and

$$\overline{z^2}(t)/\Lambda^2 \simeq t^2/T^2 \quad \text{for small} \quad t \tag{18}$$

These expressions are the asymptotes of the universal diffusion expression. They are shown in Figure 1, together with the experimental points obtained as described in the previous section. Each of these points actually represents a series of points fitting nicely into the curve toward lower values of time, and they represent the points as far away from the chimney as the smoke could be traced with reasonable accuracy. It is seen that at least an order-of-magnitude agreement is obtained, which is all that could be expected since the theoretical picture is developed under the assumption of homogeneous, isotropic, fully developed, and stationary turbulence.

Non-Gaussian cases. Non-Gaussian wind velocity fluctuation distributions were observed at Risø in January 1960 and January 1961. The distributions of the vertical component of the wind velocity observed in the morning of January 7, 1960, are shown in Figure 2. It is seen from this figure that at 8 and 9 o'clock the distributions are compatible within our resolution with a 12 per cent Gaussian plus 88 per cent δ-function distribution. The actual wind velocity recorded in time shows steady nonfluctuating sections around $w = 0$ interrupted by fluctuating sections corresponding to the Gaussian part of the distribution. We interpret this picture as meaning that the atmosphere consists of laminar regions interrupted by turbulent regions similar to the situation in fluid experiments in the transition region between laminar and turbulent flow [*Rotta*, 1959]. How-

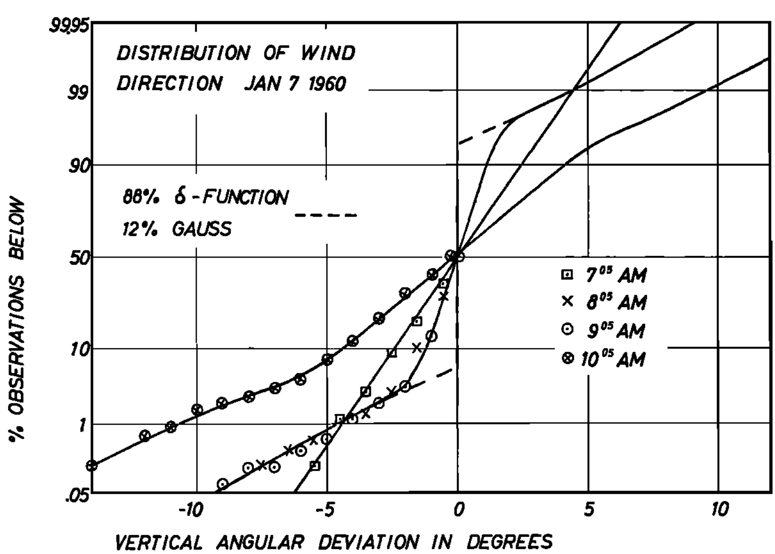

Fig. 2. Probability that the vertical angular deviation from the mean is smaller than a given value plotted against the deviation on graph paper resulting in straight-line plots if the probability distribution is Gaussian.

ever, we are unable to estimate a magnitude like Reynolds number for these atmospheric events in support of this picture. The mean wind speed at the 56-meter station where the distributions in Figure 2 were obtained was around 6 m/sec.

At 1134 on the same morning radiosonde measurements at Kastrup showed definite temperature inversions, and inversions were also recorded in the tower measurements at Risø at all times during the morning.

From the recorded track the mean length D and the volume fraction $s = 0.12$ of the turbulent regions can be obtained. Besides these parameters the parameter mean wind speed $\bar{U}$, mean square deviation $w'^2 = \overline{w^2}$, and mean square deviation $\sigma_{1T'}{}^2$ of mean wind speed when observed over limited periods of time T' can be determined.

The vertical component of the wind velocity fluctuation is observed only at the 56-meter station. Horizontal wind speeds are observed at all stations, and, assuming the same behavior of vertical and horizontal patterns, the important parameters can be extrapolated beyond the top station at 123 meters to the smoke height 150 meters, to be discussed presently.

The smoke pattern from Kyndby is illustrated in Figure 3, a photograph obtained at 0930 AM. The optical density and the total amount of smoke emitted together with the smoke temperature are measured at Kyndby. On the day in question 28 per cent of the smoke was emitted at 180°C and the remainder at 150°C. It is indeed remarkable that this division of the smoke into two components is visible over a considerable part of the smoke path, which, as seen in Figure 3, extends for approximately 20 km.

The extrapolated mean wind speed at 150 meters (the mean smoke height) was 7 m/sec, and the path thus corresponds to diffusion occurring for a period of 2900 seconds. Even then the corresponding value of $\sqrt{\overline{z^2}}$ is only 7 meters.

Figures 1 and 2 represent the most marked

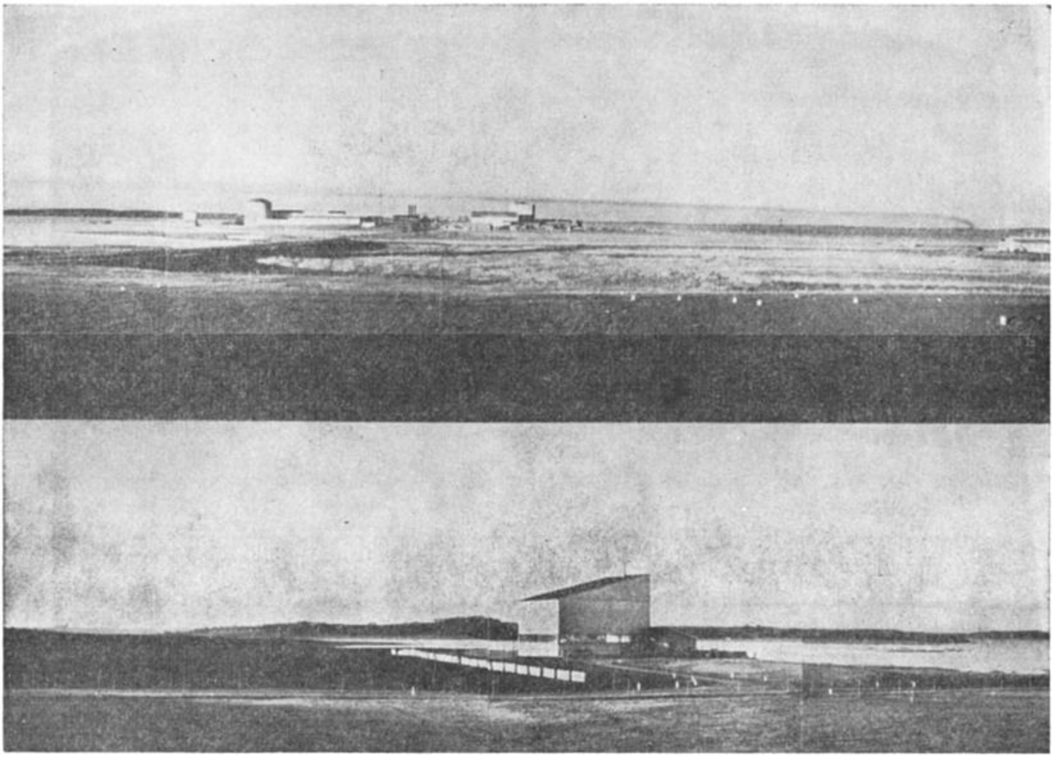

Fig. 3. Smoke plume from the chimneys of the Kyndby power station drifting 20 km past the cathedral of Roskilde as seen from Risø. In the center is the micrometeorology tower at Risø.

non-Gaussian distributions and slow diffusion observed at Risø so far. This year a case of $s = 0.4$ has been recorded; also in this occurrence the diffusion was slow.

Discussion. Various explanations could be suggested for the slow diffusion described in the preceding section, especially because of the necessary extrapolations from the 56-meter observation point to the 150-meter smoke height. Thus we could assume that on the day in question the flow pattern at 150 meters was entirely laminar or that the fluctuations at this height were of a nearly periodic structure. Also the initial cloud dimensions may have been much larger than the significant length scale of the turbulence [*Corrsin*, 1959], in which event apparently slow diffusion may result. At any rate, the observations clearly show incompatibility with the assumptions of homogeneous, isotropic, fully developed, and stationary turbulence, and the results also indicate that the validity of transformations 5 and 6 is limited to situations in which these assumptions are reasonably well fulfilled. This follows from the fact that our tower measurements would lead to a prediction of $\sqrt{\overline{z^2}} = 50$ meters instead of the measured value 7 meters.

It is interesting to note that, in homogeneous, isotropic, fully developed, and stationary turbulence, diffusion is limited to the power-law region of $\overline{z^2} \propto t^3$ to $\overline{z^2} \propto t$. This follows from equations 3, 4, and 6 together with the relations between $E_\perp(k)$ and $E_\parallel(k)$[6]. To obtain slower than Fickian diffusion from equation 3 it is necessary that $P_L(\omega)$ have an extended region of positive derivative. Thus if in the low-frequency region $P_L(\omega)$ has the form

$$P_L(\omega) \cong [g + (1 - g)(1 - e^{-\omega/\omega_0})]P_0$$

Fickian diffusion will occur with a diffusion constant reduced by the factor g and the diffusion will be described by an initial $\overline{z^2} \propto \overline{t^2}$ behavior followed by power laws in the region toward $\overline{z^2} \propto \log t$ until eventually the Fickian diffusion takes over. It is possible to fit the data we have obtained so far by assuming that g is a function of the volume ratio of turbulence s and that the cutoff frequency ω_0 is related to w' and the average size D of the turbulent regions, but such a procedure cannot be justified by theory or sufficient experimental material.

Conclusions. The theoretical transformation derived by C. F. Wandel and the author for the connection between Lagrangian and Eulerian descriptions of turbulence has been checked experimentally. Furthermore, it has been shown that the validity of the theory is limited to wind fluctuation distributions reasonably well described as Gaussian. However, such situations are by far the most frequent in the natural wind. Only the fundamental aspects of these investigations are discussed. Further experimental checks on the ideas are needed as well as a major investigation to obtain fallout patterns and dose curves if the ideas presented are to be applied to safety assessments in connection with atomic energy.

The idea of a cutoff connected with the distance D may be invoked to describe diffusion slower than Fickian and also for the diffusion of a single puff of smoke of diameter $D(t)$. Using the above formalism on the latter case leads in the Gaussian situation to diffusion patterns in the range $\overline{z^2}(t) \propto t^3$ to $\overline{z^2}(t) \propto (\log t)$ in agreement with experimental observations.

A more detailed paper will be given as a *Risø Report.*

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