Some Optical Properties of Turbulence in Stratified Flow near the Ground[1]

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Abstract. Optical scintillation was measured for horizontal paths 120 to 600 meters long, about 1.5 meters above uniform surfaces, both snow and grass. The results are related to temperature profiles, wind speeds, Richardson numbers, and path lengths. Power spectra of scintillation measured under various conditions of stably stratified flow are presented and discussed.

INTRODUCTION

Variations in density associated with turbulence produce distortions in wave phenomena propagating through the atmosphere. The distortions may be described as perturbations in both the amplitude and the phase of the incident wave. In the field of optics, the effects of perturbations have been termed scintillations by the astronomers, who, since the time of the first telescopes, have been plagued with erratic stellar images caused by atmospheric turbulence. In ordinary optical work the distortions of amplitude and phase appear as fluctuations in brightness and color and are sometimes referred to as brightness scintillation and color scintillation [*Ellison,* 1954], respectively. In addition, the distortion and associated image motion are often referred to as shimmer or 'boil' [*Middleton,* 1952].

The distortions are caused by variations in speed of propagation through turbulent elements having different densities and hence different indexes of refraction. For optical wavelengths and normal atmospheric pressures, temperatures, and constituents, it can be readily shown [*Bellaire and Elder,* 1960] that temperature fluctuation in turbulent motion is the only significant factor in creating scintillation effects.

This article presents some results of measurements of optical scintillation made during an investigation of the dependence of visual resolution on wind and temperature conditions near the ground. Constant-intensity, parallel-beam light sources were placed at several distances up to 600 meters from a telephotometer. Both the light sources and the telephotometer were about 1.5 meters above uniform and horizontal grass and snow surfaces. The measurements were similar to those described by *Siedentopf and Wisshak* [1948], *Tatarski, Gurvich, Kallistratova, and Terenteva* [1958], and *Gurvich, Tatarski, and Cvang* [1958]. Related findings have been reported by a number of investigators in the United States also, none of whom, however, as far as is known, has included appropriate micrometeorological measurements with which the observed scintillation characteristics can be related to flow properties. The results described here, furthermore, cover a wider range of stratification than has been reported previously.

Most theoretical analyses of the subject have been made for stellar scintillation for both optical and radio wavelengths. The works of *Rayleigh* [1893], *Bergmann* [1946], *Booker, Ratcliffe, and Shinn* [1950], *Hewish* [1951], *Chandrasekhar* [1952], *Fejer* [1953], *van Isacker* [1954], *Keller and others* [1956], *and Silverman* [1956] should be mentioned. A comprehensive survey of much of the work in this area is given by *Nettlebad* [1953]. More recently, analytical models appropriate for horizontal optical paths in the surface layer of the atmosphere have been developed by *Obukhov* [1953], *Chernov* [1960], and *Tatarski* [1961]. In the last both theory and experimental results are given for propagation near the ground.

Tatarski's development is based on the first-

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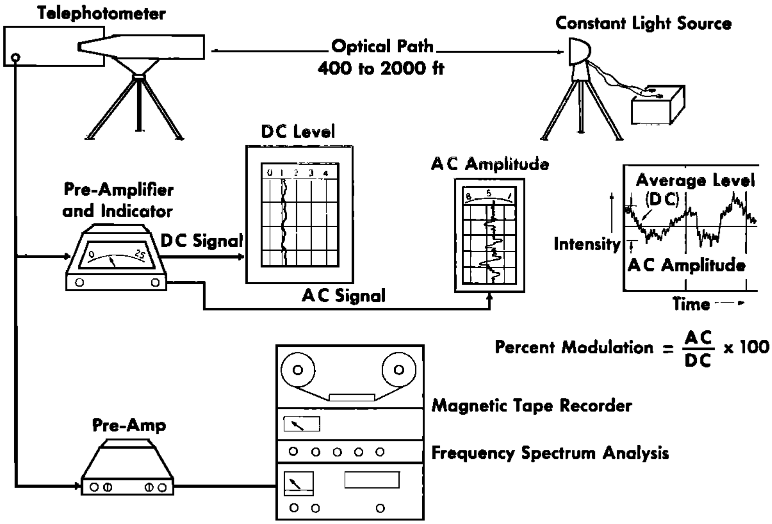

Fig. 1. Block diagram of scintillation measurement system.

order perturbation theory of wave propagation in an isotropic turbulent field in which the index of refraction may be regarded as a conservative passive additive. The specific quantitative analysis of direct concern depends on the assumption that the structure function of the index of refraction is proportional to the ⅔ power of the separation distance in accordance with the Kolmogorov equilibrium range. The light source and most of the path lengths employed for the findings reported here are such that Tatarski's theory for the condition

$$l_0 \ll (\lambda L)^{1/2} \ll L_0$$

would be valid. Parameter l_0 is the microscale of turbulence, L_0 the external scale, λ the wavelength of light, and L the length of the optical path. In the presentation that follows frequent reference is made to Tatarski's theoretical analysis and to the experimental findings reported by him and his colleagues.

Measurement Location and Instrumentation

Measurements were made over snow on an airfield in northern Michigan and over grass on one in southern Michigan. The optical path was about 1.5 meters above each surface, and the area upwind was uniform and level for at least 250 meters. The optical path for the snow measurements was 550 meters long, and that for the grass measurements varied from 122 to 600 meters.

The general scheme of scintillation measurement is shown in Figure 1. The light sources were 5-inch sealed-beam spot lamps powered by 12-volt batteries. The telephotometer had a 3-inch aperture and a lens system that focused the light on a Dumont type 6467 multiplier phototube. The intensity of fluctuation was measured in terms of the per cent modulation of the received signal, defined as the ratio of the mean peak-to-peak amplitude to the average, or d-c, level. This quantity, *Pm*, gives a measure of the intensity of fluctuations independent of changes in brightness due to attenuation by the atmosphere. For spectral analysis the phototube signal was also recorded on magnetic tape by means of frequency modulation of a carrier signal.

Continuous records of meteorological variables were made throughout all measurement periods. Wind and temperature profiles were measured with sensors at 0.5, 1, 2, and 4 meters above the surface at a single location near the optical path. The sensor array is shown in Figure 2. The anemometers were standard Beckman and Whitley instruments matched to within 1 per cent. The temperature sensors were copper-constantan thermocouple junctions (less than 0.2-mm diameter) supported in flat-plate radiation shields. The recording systems permitted

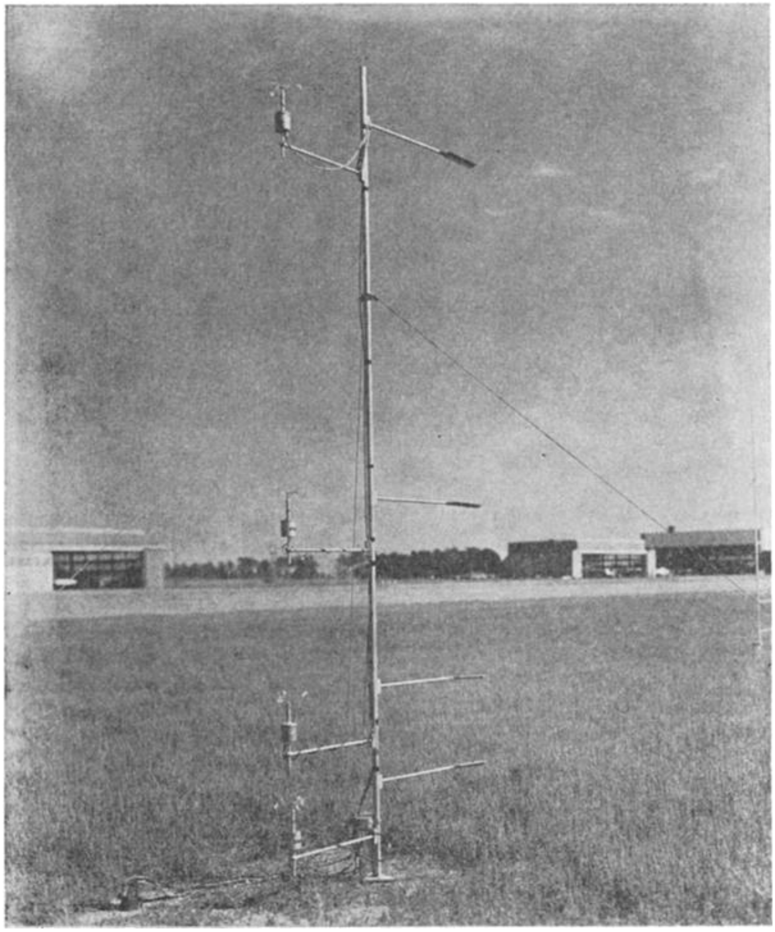

Fig. 2. Micrometeorological profile mast with anemometers and shielded thermocouples at four heights.

resolution of about 0.1°C temperature difference and 1.5 meters of air movement. Wind direction was measured with a single vane at a height of about 4 meters.

Results and Discussion

Per cent modulation and thermal stability. If the influence of pressure and water-vapor fluctuations are negligible, direct dependence of scintillation on temperature fluctuations may be expected. Thus, for turbulent flow statistically homogeneous in the horizontal, the average intensity of scintillation should be directly related to the (mean, vertical) temperature profile. *Siedentopf and Wisshak* [1948] measured scintillation, wind speed, and solar radiation simultaneously, finding that maximum per cent modulation coincided with maximum radiation received and that a secondary maximum occurred during the night with nocturnal radiation loss. A minimum in per cent modulation occurred shortly after sunrise and again before sunset. Although they did not measure temperature profiles, the results led them to conclude that scintillation was closely associated with the magnitude of the mean temperature gradient normal to the optical path.

Tatarski, Gurvich, Kallistratova, and Terenteva [1958] measured the intensity of scintillation for optical paths varying from 550 to more than 3000 meters at a height of 1 meter over a flat region of the steppe. Their scintillation measurements were in terms of the root-mean-square fluctuations of light in-

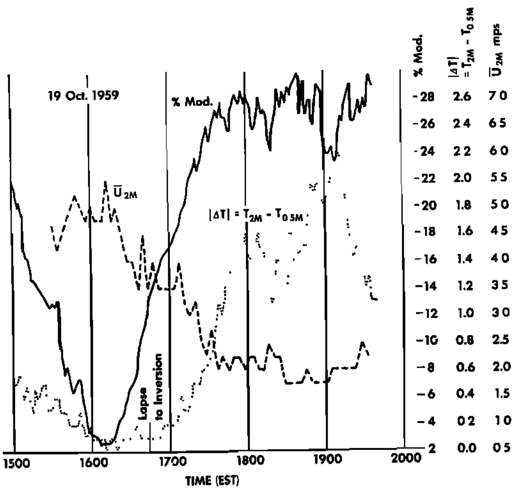

Fig. 3. Typical variation of scintillation through a sunset period.

tensity, and they found that this parameter varied as the 0.35 power of the temperature gradient with a correlation coefficient 0.9. The relationship was independent of the length of path.

Figure 3 presents some typical findings in the present investigation. It shows the variation of per cent modulation through a cloudless sunset period in mid-October. The measurements were made over grass for a 600-meter path. Simultaneous measurements of average temperature difference between 2 meters and 0.5 meter and average wind speed at 2 meters are also shown. The temperature difference varied from a midafternoon lapse value of about 0.5°C, through an adiabatic period before sunset (1715 EST), to a nighttime inversion which occasionally reached 2.2°C. The 2-meter wind speed varied between 3 and 6 mps during the lapse period and between 2 and 3 mps during the inversion period.

The most striking feature of the scintillation pattern is its correspondence with the absolute value of vertical temperature difference. A broad daytime maximum (not shown) occurred a short time after noon, and, as the lapse rate decreased with decreased solar heating, scintillation dropped to a distinct minimum before sunset at the time of near-adiabatic conditions. The wind speed was about 4 mps during the near-adiabatic period, and turbulent flow would be expected, but, since vertical motions through the adiabatic layer could create no temperature discontinuities, the measured per cent modulation at its minimum value must be accredited to pressure and water-vapor fluctuations or to the noise level of the measurement system.

The above reasoning does not account for the increase in scintillation between 1615 and 1700 EST when the temperature difference remained near an adiabatic value. Similar characteristics have been observed on several occasions and are due, apparently, to radiation error in temperature measurement. At low sun angles the flat-plate radiation shields were ineffective and, because of the wind speed increase with height, the lower probes could indicate temperature significantly greater than true.

During the night the scintillation showed a second maximum in the presence of the nocturnal inversion. This period was characterized by frequent changes in temperature difference caused by light and variable wind. The resulting variations in scintillation intensity were marked by brief intervals of extreme shimmer and scintillation when increases in wind speed mixed highly stratified layers.

A systematic relationship between scintillation intensity and average vertical temperature difference was found in the measurements made over snow in inversion conditions. The results for a large number of 2-minute averages were grouped and plotted in logarithmic coordinates as shown in Figure 4. Each point is the mean

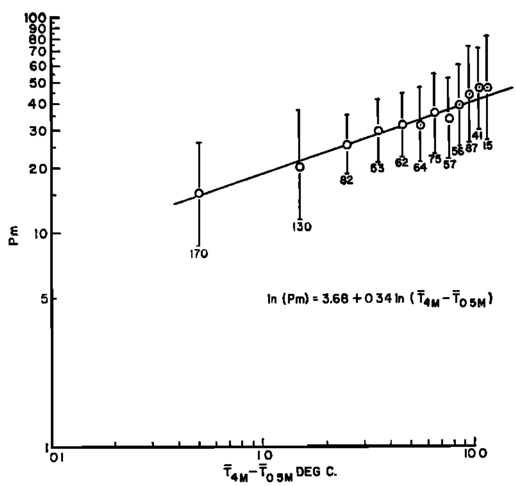

Fig. 4. Per cent modulation versus temperature difference between 4 meters and 0.5 meter over snow.

of the indicated number of averages of per cent modulation within 1-degree temperature difference increments. The vertical line segment centered on each point indicates the standard deviation as computed on a logarithmic scale.

The grouped data were subjected to a weighted linear regression analysis. The resulting line, as shown in the figure, has a slope of +0.35. The fact that this figure is not different from that found by *Tatarski, Gurvich, Kallistratova, and Terenteva* [1958] must be regarded as fortuitous, since they related the root-mean-square fluctuations[2] to temperature differences between the heights of 8 and 2 meters. Their optical path was about 1 meter above the surface. The results of both investigations, however, confirm the obvious significance of the mean temperature gradient in controlling the intensity of scintillation.

The scintillation data used for Figure 4, regrouped according to both temperature difference and wind speed, after some smoothing, produced the pattern shown in Figure 5. It is apparent that, for a constant temperature difference, an increase in wind speed causes an increase in scintillation. An exception seems to exist for per cent modulation greater than 50, where a decrease occurs for wind greater than 1.5 to 2 m/sec. The relatively few data in this category, however, may be misleading.

The systematic relationships shown in Figures 4 and 5 suggest the Richardson number as a useful parameter for characterizing scintillation conditions. Since scintillation can be expected to depend on both the magnitude of the temperature gradient and the intensity of vertical motions, the ratios of per cent modulation to mean temperature difference were plotted against Richardson numbers in logarithmic coordinates as shown in Figure 6.

'Quasi-local' Richardson numbers were calculated from 1- and 2-meter data according to Lettau's definition [*Lettau and Davidson*, 1957]. As in Figure 4, the number of 2-minute averages included in each point is indicated

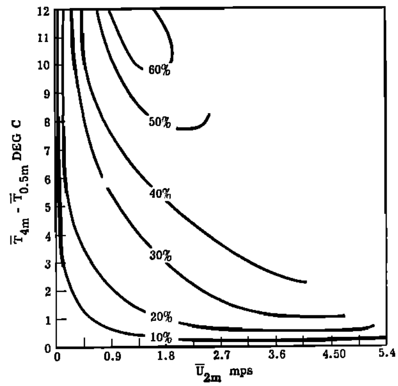

Fig. 5. Relationship of per cent modulation to temperature difference between 4 meters and 0.5 meter and wind speed at 2 meters over snow.

along with the standard deviation computed on a logarithmic scale. The grouping was arbitrarily based on equal intervals of Richardson numbers on the logarithmic scale. The two lines whose equations are shown in the figure were obtained by linear regression with each point weighted according to the number of observations averaged. The best point of intersection was determined by trial. The choice of two straight lines instead of a continuous function was arbitrary and was based on the idea of the existence of a critical Richardson number separating laminar from turbulent flow.

The intersection of the two lines suggests a change in average flow characteristics such that the decrease in amount of scintillation per unit temperature gradient with increasing stability is nearly an order of magnitude greater for $Ri < +0.35$ than for $Ri > +0.35$. For purely laminar flow it may be argued that scintillation intensity would be constant and at a minimum similar to that observed during adiabatic conditions. Any relationship between the parameter $Pm/\Delta T$ and Ri for Ri greater than critical would represent merely how the temperature difference, alone, varied with Richardson number. For the region $+0.35 < Ri < +10$, however, there was a relatively large scatter in the original data, as shown by the standard deviations, and it was the experience of the observer that, during the conditions represented here, both visual resolution and scintillation varied between wide limits. Small increases in wind speed were observed to cause very high scintillation for short periods following relatively

[2] The results of these authors are given in terms of the quantity

$$\sigma^2 = \overline{(I - I_0)^2}$$

in which I is the apparent intensity of the light source. It can be shown that σ is related to Pm by a constant factor for Pm significantly less than 100.

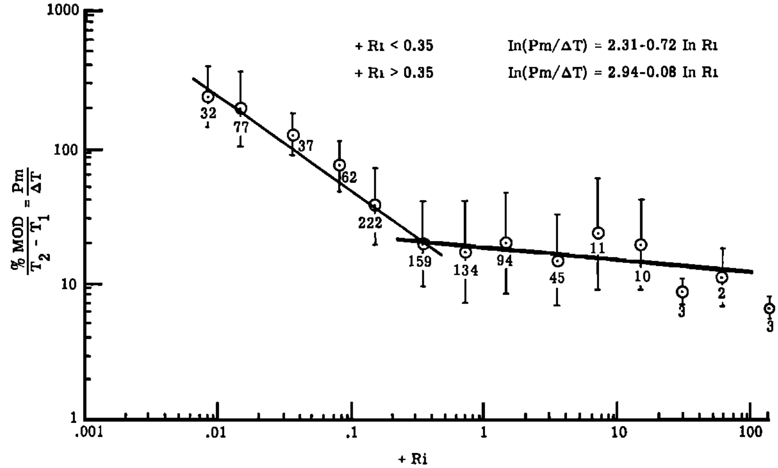

Fig. 6. Per cent modulation per unit temperature difference between 1 and 2 meters versus Richardson number over snow.

longer intervals of extreme stratification and almost no scintillation. The observations support the idea of the presence of internal gravity waves with frequent breaking to brief periods of turbulent flow. The averages of such data make up the points in Figure 6 for $+0.35 < Ri < +10.0$, so that this region may be regarded as one characterized by intermittent turbulence causing extreme scintillation for short intervals.

Per cent modulation and path length. Although it is commonly observed that the optical effects of turbulence near the ground increase with increase in path length, apparently no quantitative observations of length-of-path dependence had been made before those of *Siedentopf and Wisshak* [1948]. They found that the relationship between per cent modulation and length of path had the characteristics of a saturation curve that approached its limiting value at a path length of 1200 meters. *Bellaire and Ryznar* [1961] have shown that the data of Siedentopf and Wisshak may be represented, however, by separate relationships according to path length, L, as follows:

$80 \text{ m} < L < 400 \text{ m}$	$Pm \propto L^{1.3}$
$400 \text{ m} < L < 1000 \text{ m}$	$Pm \propto L^{0.75}$
$1000 \text{ m} < L < 1600 \text{ m}$	$Pm \propto L^{0.10}$

Siedentopf and Wisshak made their measurements at night with a light source near the photometer directed toward a corner mirror. The mirror was placed at different distances varying from 40 to 800 meters. The scintillation conditions were reported to be steady, but no information on wind and temperature conditions was given.

Gurvich, Tatarski, and Cvang [1958] (see, also, *Tatarski* [1961]) have presented theoretical and experimental results that correspond, respectively, to $Pm \propto L^{0.91}$ and $Pm \propto L^{0.98}$. As was noted earlier, the theory is based on the ⅔ law for the condition

$$l_0 \ll (\lambda L)^{1/2} \ll L_0$$

Their data were obtained by placing a light source at various distances from 250 to 2000 meters, and although different heights were used for paths of different lengths the data were adjusted according to measured temperature gradients.

Some results of the present investigation are shown in Figure 7. The measurements were made with separate light sources placed at 122, 244, 366, 488, and 610 meters from the telephotometer. A remote relay switching arrangement permitted the light sources to be activated in rapid sequence so that the effects of varying wind and temperature conditions could be minimized. For each of the experiments the wind speed was about 3 mps, but the direction relative to the path varied from $\alpha = 0°$ to $\alpha = 090°$, as noted in the figure.

From the tabulations in the figure it is seen that the data for tests 2 and 3, having temperature differences of $-0.25°$C and $+0.20°$C, re-

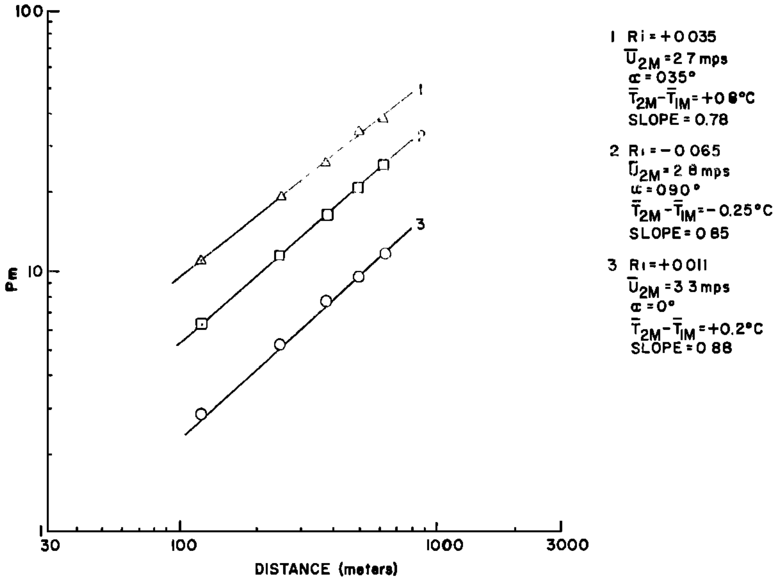

Fig. 7. Per cent modulation versus length of optical path.

spectively, give exponents for L not greatly different from that (0.91) predicted by Tatarski, whereas those for test 1, with a temperature difference of +0.8°C, yield a value of only 0.78. Possibly deviation from isotropy and the ⅔ law in the stably stratified flow is responsible for the lack of closer agreement.

Scintillation power spectra. A major result of *Tatarski*'s [1961] theoretical development for scintillation is that the normalized frequency spectrum depends only on the quantity $f(\lambda L)^{1/2}/v_n$, in which f is the frequency (cps), λ the wavelength of light, L the optical path length, and v_n the cross-wind (to the path) component of the wind speed.

The theory is based on 'frozen-in' turbulence, i.e., turbulence in which the discontinuities themselves do not change as they move through the optical path. It is valid, furthermore, only when the angle between the wind direction and the optical path satisfies the inequality

$$v_n/v_t \gg (\lambda/L)^{1/2}$$

in which v_n and v_t are the wind components normal to and parallel to the optical path, respectively.

To account for the 'aperture smoothing' effect of a receiver, Tatarski develops the theory further to include a term that decreases the spectral energy and shifts the maximum toward lower frequencies as the aperture diameter is increased.

Since the scintillation measurements reported here were obtained with a 3-inch aperture, an exploratory test was made to determine experimentally the effect of aperture size on spectral measurements. An adjustable aperture was fitted to the telephotometer, and successive 2-minute observations were made with apertures 3, 2.5, 3, 2, 3, 1.5, 3, 1, 3, 0.5, and 3 inches in diameter. The observations were made at night over a short grass surface with an optical path about 550 meters long. Wind and temperature conditions were unusually steady throughout the experiment; the alternating measurements with the 3-inch aperture showed relatively minor variations from one to the other.

The results are shown in Figure 8, in which the square root of normalized power per unit frequency is plotted against frequency. There is relatively little difference between data from the 3- and the 2.5-inch apertures. As the aperture is decreased to 1 and 0.5 inch, however, the relative power below 20 cycles decreases and that above increases. The total power before normalization increased from a per cent modulation of 31 at 3-inch aperture to 74 at 0.5-inch. The results are in general agreement with Tatarski's theoretical analysis.

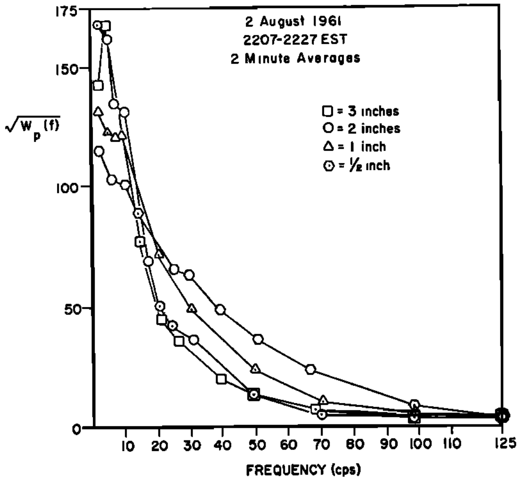

Fig. 8. Effect of aperture size on normalized frequency spectra of scintillation.

The spectral data shown in Figure 8, as well as those described below, were obtained directly from the (frequency modulation) analog recording on magnetic tape. The original signal for a 2-minute interval was re-recorded on a 75-foot tape loop that could be replayed at various speeds on a tape loop machine capable of a factor-of-8 time compression. The signal was demodulated, and its analog was supplied to a wave analyzer. The system permitted measurement of the rms amplitude in any 1.25-cps band over the range of 2.5 to 2500 cps. A similar method of spectral analysis has been described by *Parks* [1960].

Spectral analyses were made for eight 2-minute periods of scintillation recording obtained in inversion conditions over snow. Pertinent meteorological and scintillation data are listed in Table 1. The periods were selected to cover a wide range of visual resolution conditions, and they represent independent observations insofar as all except periods I and III were measured on separate days. These two were recorded on the same day but were more than 30 minutes apart. Results of the analyses are presented in three different ways in Figures 9, 10, and 11. Each of the three figures shows all eight spectra, but the different representations facilitate comparison of the spectra in relation to flow properties.

In Figure 9, the spectra are shown as measured. The most striking differences among the spectra appear in the total power represented by

$$\int_{2.5}^{200} W_p(f)\, df$$

for the different curves. The integrals should be proportional to the per cent modulation values measured simultaneously. A comparison shows reasonable agreement for periods I through VI. The integrals for periods VII and VIII are too small in relation to the measured per cent modulation values. This discrepancy may be traced to possible frequency dependency of the per cent modulation recording circuit, a matter that is currently under investigation.

For more direct comparison the spectra were normalized by a factor to render all eight integrals equivalent. It was found, then, that the spectra could be grouped arbitrarily into four categories according to general shape characteristics. These are shown in Figure 10 along with appropriate average Richardson numbers and cross-wind components for the groupings. The spectra in group 1 represent periods for which $Ri > +0.35$, the critical value suggested by Figure 6. The high values of relative power

TABLE 1

Period	$T_{2m} - T_{1m}$, °C	$\bar{u}_{1.5m}$, cm sec^{-1}	u_n, cm sec^{-1}	Ri	% Modulation	f_m	m
I	1.7	137	69	+0.19	62	11	0.26
II	2.0	53	51	+0.60	62	9	0.31
III	1.3	94	60	+0.16	39	10	0.28
IV	0.9	310	131	+0.13	40	12	0.15
V	0.8	72	46	+3.77	28	7	0.24
VI	1.5	96	93	+0.59	29	8	0.15
VII	0.05	462	264	+0.01	11	27	0.17
VIII	0.4	146	112	+0.35	13	12	0.18

for the low frequencies, in comparison, for example, with the spectra in group 3, which represent periods of comparable cross wind, support the idea that internal gravity waves were responsible for much of the scintillation observed during these periods.

The effect of cross-wind speed should be seen by comparing groups 3 and 4, which have comparable Richardson numbers but cross-wind speeds that differ by a factor of 2. Although these two groups are quite similar, a small shift in relative power to higher frequencies for the higher cross-wind speed may be noted. Between 7.5 and 35 cps the relative power in group 4 is consistently higher than that for group 3, with the opposite trend below 5 cps. It should be noted that Ri for period VIII in group 4 was +0.35. This may be responsible for the relatively high power indicated at 5 cps for this period. It is not clear why the relative power at 40 cps is higher for group 3 than for group 4.

The single spectrum identified as group 2 for convenience represents a period of small Ri (+0.01) and relatively high cross wind. An expected decrease in power for the lower frequencies is clearly evident.

In Figure 11 the eight spectra are shown in terms of the product $fW_p(f)$, with frequency on a logarithmic scale. This representation makes it possible to compare spectra conveniently in terms of the frequency of maximum value of $fW_p(f)$. Thus, to compare these results with those obtained by *Tatarski* [1961] the maxima, f_m, have been computed and listed in Table 1. Following Tatarski, f_m is defined as the average of the two frequencies at which $fW_p(f)$ is one-half of the maximum measured value. To eliminate the influence of normal wind speed, path length, and wavelength of light, the parameter

$$m = f_m(\lambda L)^{1/2}/v_n$$

was also computed for each period and listed in Table 1. The values range from 0.15 to 0.31, with an average of 0.22.

Tatarski's theoretical value for m, postulated for an infinitely small aperture, is 0.55. He found, however, an average experimental value of 0.32 for 80 spectra obtained with a 2-mm aperture. It is not surprising, therefore, to find an average m of 0.22 for these data obtained with a 3-inch aperture.

There appears to be little correlation between m and Ri, although there is a suggestion of correlation between f_m and Ri. For the three cases with $Ri > +0.35$, f_m averages about 8; for those with $0.1 < Ri < 0.35$, f_m average is about 11; and for the single case with $Ri = 0.01$, $f_m = 27$. It may be concluded, tentatively, that f_m decreases with increasing Richardson number. It is not clear, however, why normalization to cross-wind speed degrades rather than improves the correlation. One possible cause may be the inadequacy of the wind direction measurements, since they were obtained at a single location near the optical path. With light winds it is likely that this single measurement is quite inaccurate for time intervals as short as 2 minutes.

The spectral data presented here must be regarded as a limited sample of scintillation characteristics possible under a wide range of stable conditions over snow. Spectral analysis of additional data is currently under way to explore further relationships between spectral characteristics and flow conditions.

Summary and Conclusions

The average intensity of optical scintillation over a horizontal path in turbulent flow near the ground was found to increase systematically with increase in absolute magnitude of the mean vertical temperature gradient. For stable conditions over snow a larger number of observations indicated that, for a given temperature gradient, scintillation intensity increases with increasing wind speed. The snow data were normalized with respect to mean temperature gradient, and it was found that resulting ratios decreased systematically with increasing (positive) Richardson numbers. A change in slope at $Ri = +0.35$ and other optical evidence suggest that for $Ri > 0.35$ much of the scintillation may have been due to turbulence associated with the breaking of internal gravity waves.

For three separate experiments it was found that the average intensity of scintillation was proportional to the length of path raised to an exponent, the experimental values of which were 0.88, 0.85, and 0.75. Because these values decrease with increasing absolute value of the temperature gradient, the difference between them and the theoretical value of 0.91 predicted by Tatarski for isotropic turbulence in

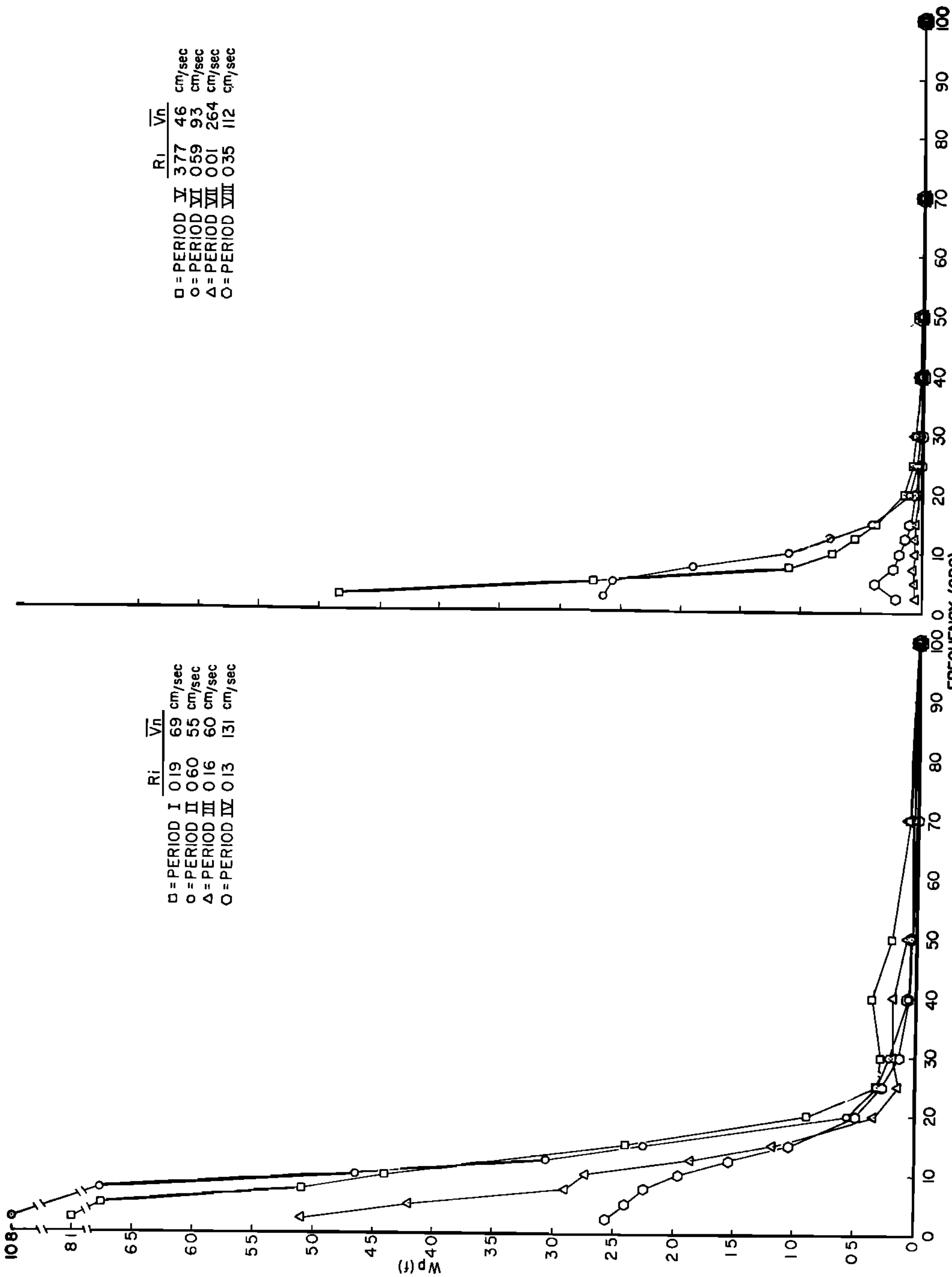

Fig. 9. Power spectra of scintillation for stable conditions over snow for eight periods identified in Table 1.

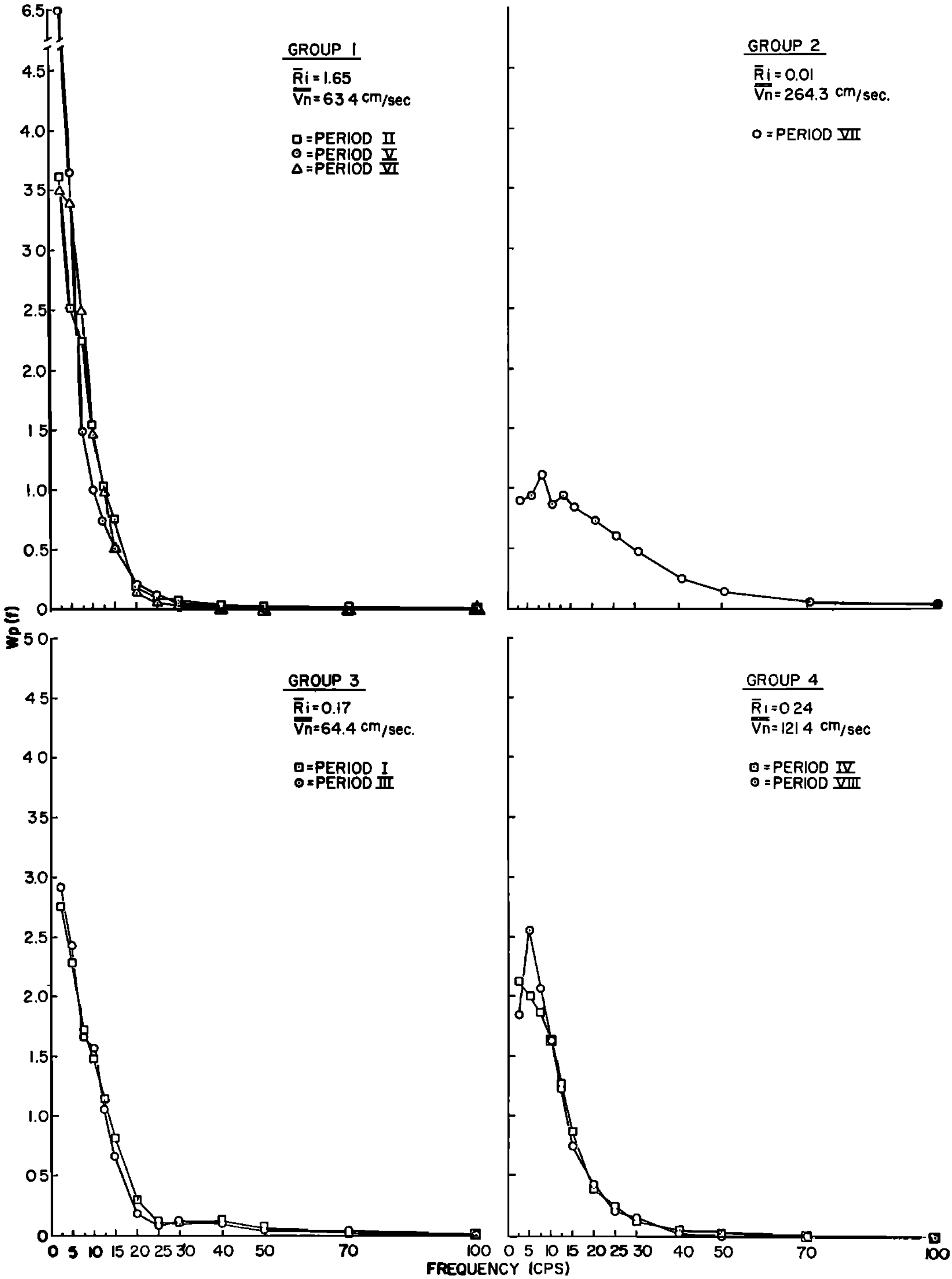

Fig. 10. Normalized power spectra of scintillation for stable conditions over snow for eight periods identified in Table 1. (Power per unit frequency.)

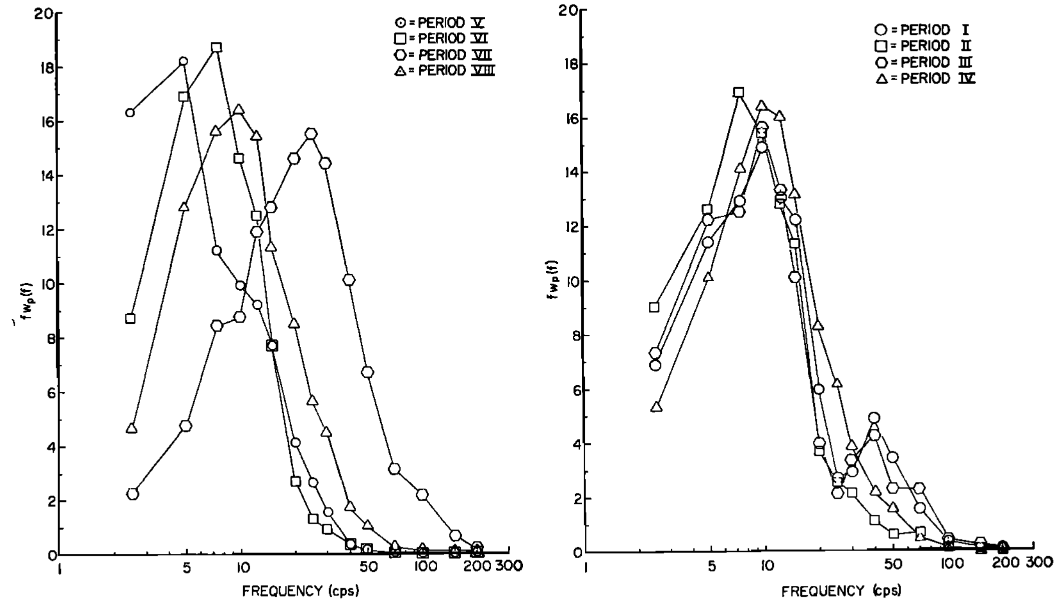

Fig. 11. Normalized power spectra of scintillation for stable conditions over snow for eight periods identified in Table 1. (Power per logarithmic frequency increment.)

which the ⅔ law holds may be taken as a measure of deviation from these conditions due to buoyancy effects.

Spectral analysis of eight separate 2-minute scintillation recordings in stable stratification over snow showed a tendency for low frequencies to dominate for conditions with high Richardson numbers. An expected improvement in correlation between stability and spectral characteristics after normalization to wind-speed component perpendicular to the optical path was not apparent. Because wind was measured at only a single location near the optical path it is likely that the computed wind-speed components were not representative of conditions over the entire path so that expected improvement in correlation was obscured.

A test of the effect of the size of the receiver aperture on spectral results showed (in accordance with Tatarski's theoretical result) a shift in relative power from low to higher frequencies as the aperture diameter was decreased.

Both the size of the aperture and the relatively strong stability during these measurements may account for the fact that the normalized spectral maxima $m = f_m (\lambda L)^{1/2}/v_n$ ranged from 0.15 to 0.31 in comparison with Tatarski's theoretical value of 0.55 for an infinitely small aperture. Additional spectral analyses and aperture tests are required to establish reliable relationships between scintillation and turbulence in stratified flow.

The results reported here represent preliminary findings of a natural phenomenon fairly common in the earth's atmosphere. Analysis in terms of thermal stability, wind speed, and Richardson number permits application to various scientific and engineering problems involving electromagnetic propagation through the atmosphere. In addition, there exists the possibility of using optical scintillation to study average atmospheric turbulence characteristics in long paths over inaccessible areas such as forests, glaciers, or water surfaces.

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